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                    FOREST SERVICE HANDBOOK
                        WASHINGTON
FSH 2409.12a - TIMBER VOLUME ESTIMATOR HANDBOOK
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2409.12a - Establishes new Timber Volume Estimator Handbook which
provides Service-wide standards and instructions for preparation of
equations or tables used to estimate the timber content of trees.
F. DALE ROBERTSON

Chief

# UNITED STATES DEPARTMENT OF AGRICULTURE FOREST SERVICE 

## FSH $2409.12 a$

TIMBER VOLUME ESTIMATOR HANDBOOK

## UNITED STATES DEPARTMENT OF AGRICULTURE

 FOREST SERVICETIMBER VOLUME ESTIMATOR HANDBOOK

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TIMBER VOLUME ESTIMATOR HANDBOOK

## ZERO CODE

This handbook provides guidance for preparing highly complex mathematical representations of trees and standard procedures for producing, selecting, and using tree volume estimators. The intended audience of this text is Region and Station specialists responsible for timber measurements, contractors developing volume estimators and associated software, and individuals involved in collecting and maintaining felled tree information. The procedures in this handbook relate to the measurement of trees using cubic feet or board feet measure.

01 - AUTHORITY. Under the rules at section 223.36 of Title 36 of the Code of Federal Regulations (36 CFR 223.36), the Forest Service may sell timber based on cubic volume, board foot volume, or weight.

02 - OBJECTIVE. To compile and maintain consistent, standardized tree measurement records that are suitable for preparing tree volume equations.

03 - POLICY. When measuring felled trees or standing trees using precision optical devices, follow the minimum data collection standards of this section.

1. When measuring felled or standing trees for the purpose of creating volume equations:
a. Use the minimum data collection standard specified in sections 11-15.35c
b. Store in a local data base that meets the specified standards stated in section 21.
c. Manage the data with multiple users in mind.
d. Use the technique of stem profile equations (chapters 30 through 60) under all but the most unusual circumstances.
2. All resource functions which need estimates of tree volume should use the same estimators and computer software to the extent possible. Use the same tree volume estimators for timber inventories, land management plans, timber surveys, timber sales, silvicultural examinations, and growth and yield models.
3. Use estimators that have the ability to provide volume and other product estimates at different merchantability specifications, top diameters, and log lengths, and that include options that simulate actual measurement practices applied in the general geographic area as appropriate. Use the same measurement technique for inventory volumes, planned sell, and harvest figures.
4. Apply the appropriate scaling rules when developing merchantable volume estimators. Use the Cubic Rule (Cubic Scaling Handbook, FSH 2409.11a) for determining cubic volume. Use the Scribner Decimal C or the International $1 / 4$ Inch rules (National Log Scaling Handbook. FSH 2409.11) for board foot volume determination. Use other scaling rules only when required.

04 - RESPONSIBILITY.
04.1 - Washington Office.
04.11 - Deputy Chief, National Forest System. The Deputy Chief for the National Forest System is responsible for approving scale rules used to determine timber content for volume estimators (FSM 2443.04a).
04.12 - Director, Timber Management. It is the responsibility of the Director of Timber Management to:

1. Ensure the standard procedures are updated as new technology or research improves methods for volume determination.
2. Ensure maintenance of the data bases used to store data collected for determination of volume estimators.
3. Designate a national data base coordinator.
4. Secure Washington Office approval for data elements and standards recommended by Regional Foresters and Station Directors.
5. Provide consistency in timber volume estimator data across all Regions and Research Stations.
04.13 - National Data Base Coordinator, Timber Management. It is the responsibility of the national data base coordinator to:
6. Assist field units in making changes to the structure of their data base.
7. Assist field units in storage and retrieval of data maintained in their data base.
8. Ensure the information in the data bases is in a usable state.
04.2 - Field Offices.
04.21 - Regional Foresters. It is the responsibility of the Regional Forester to:
9. Provide accurate and reliable timber volume estimators for each major timber species and area within the Region.
10. Ensure that the estimators are validated at least once every 10 -year period.
11. Ensure training of and assistance to Forest level personnel in using local timber volume estimators which meet the standards of this Handbook.
12. Ensure that those assigned to prepare volume estimators have advanced statistical (particularly applied regression analysis), mathematical, and mensurational skills (training or experience equivalent to an advanced degree in forest mensuration) needed to select and prepare appropriate timber volume estimators.
13. Ensure that all felled tree data and data collected through precise measurement of standing trees for the purpose of preparing volume estimators are taken and stored according to the standards presented in this handbook.
14. Recommend necessary additional data elements or revisions of data standards or procedures to the Washington Office Director of Timber Management.
15. Ensure preparation and maintenance of a data base for the purpose of storing and retrieving information used by the Region to prepare timber volume estimators.
16. Justify use of other than stem profile equations when new estimators are prepared.
17. Ensure consistency of use of volume estimators among all timber management functions.
18. Ensure correct application of tree volume estimators.
19. Ensure a correct segmentation rule for Forest use.
20. Approve minimum merchantability specifications for top and breast high diameters.
21. Provide direction needed to measure poles and pilings, where National Forest trees are to be appraised as poles or piling.
22. Approve Regional and local volume estimators to use for all tree species found within the Region.

### 04.22 - Station Directors. It is the responsibility of the Station

 Director to:1. Ensure that all felled tree data and data collected through precise measurement of standing trees for the purpose of preparing volume estimators are taken and stored according to the standards presented in this handbook.
2. Recommend necessary additional data elements or revisions of data standards or procedures to the Washington Office Director of Timber Management.
3. Prepare and maintain a data base for the purpose of storing and retrieving information used by the Station to prepare timber volume estimators.
4. Provide advice to mensurationists and users on technical volume estimation procedures.
04.23 - Region and Station Mensurationists. Mensurationists are responsible for choosing a technique most appropriate to the forest area being evaluated and to derive estimators for use by field units in tree volume estimation projects.

05 - DEFINITIONS.
Calibration. The process of adjusting to local conditions that are known to vary from those upon which the model was based.

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Estimation. The statistical process of deriving information about a tree as a function of the measured tree variables.

Estimators. The equations and procedures used to derive volumes.

Evaluation. Involves the consideration of how, where, and by whom the model should be used, how the model and its components operate, the quality of system design, and its biological realism.

Model Formulation. The specification of a mathematical function to be used to relate actual taper rates to measured stem data.

Validation. The testing of a model against one or more independent data sets.

Verification. The process of testing a model with data on which it was based to eliminate gaps in programming logic, flaws in algorithms, and bias in computation.

08 - REFERENCES. Users may consult the following references for additional guidance.

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Spurr, Stephen H. 1952. Forest Inventory. The Ronald Press. New York.

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11 - ORGANIZATION OF THE DATA ELEMENTS. Organize the data collected
in standardized groupings as listed in the following sections.
11.1 - Site and Location Information. This information describes the
area where the tree grew (FSH 2409.14, Sec. 71.1, 73-77). Use
information about area identifiers as well as physiographic
characteristics for identification of the data.
11.2 - Measurement Information. Record the information about the
methods used to take the measurements and to record the data.
11.3 - Tree Information. Record information that describes the
attributes of the standing tree and the area immediately surrounding
it. (FSH 2409.14 Sec 71.2).
11.4 - Segment Information. Record information about the diameters,
lengths and bark thicknesses for each of the various segments of the
tree bole. (FSH 2409.14 Sec 71.2).
12 - REQUIRED AND OPTIONAL DATA ELEMENTS. Data elements are classed
as required or optional and are listed in Exhibit 01. Required
elements are indicated by an asterisk in Exhibit 01. Exhibit 02 is a
sample field record containing the required and commonly used
optional data elements. Use this or an equivalent format which must
include the required data elements and may include additional
optional data elements.
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12 - Exhibit 01
Required (*) and Optional Data

## LOCATION INFORMATION

| 1* | region |
| :---: | :---: |
| 2* | forest number |
| 3* | district number |
| 4* | location ID number |
| 5 | state |
| 6 | county |
| 7* | data source |
| 8* | X coordinate |
| 9* | Y coordinate |
| 10* | UTM zone |
| 11* | X/Y coordinate system |
| 12* | date |
| 14 | site productivity class |
| 15 | survey cycle |
| 16 | random number, location |
| 17 | compartment number |
| 18 | stand number |
| 19 | name (comments) |
| 121* | elevation |
| 122 | ecological type (habitat type) |
| 123 | eco/habitat type reference |
| 124* | aspect |
| 125* | slope |
|  | MEASUREMENT INFORMATION |
| 13* | measurement system (Eng/Metric) |
| 105* | crown ratio method |
| 116* | dia. measurement method |
|  | TREE INFORMATION |
| 100* | sample point |
| 101* | tree number |
| 102* | plant species |
| 103* | dbh |
| 104 * | crown ratio |
| 106 | crown class |
| 107 | tree class |
| 108 | dbl bark thickness (dbh) |
| 109* | tree height (length) |
| 110* | number of cuts |
| 111 | sequence tree number |
| 112 | accumulated length |

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12 - Exhibit 01 -- Continued
Required (*) and Optional Data
```

TREE INEORMATION (cont.)

113
114
115
117 118
119
120
121*
122
123
124*
125*
126 127a* 127b*
128* 129 130
131
132
133
134
135
136

206*
$\mathrm{DIB}_{1}$
$\mathrm{DIB}_{2}$
208* height of measurement
209
210
211
record number
sawlog top diameter DIB.
17.3' Girard FC
33.6' Girard FC
assumed stump height
random number, tree
elevation
ecological type (habitat type)
eco/habitat type reference
aspect
slope
tree age
height growth
height growth interval
radial growth
basal area/acre
tree top condition
ocular tree height
percent cull
height to crown
height to major fork
site index
site species
site index reference
SEGMENT INFORMATION
segment number
segment ID (fork code)
segment length
mid height
$\mathrm{DOB}_{1}$
$\mathrm{DOB}_{1}$
$\mathrm{DIB}_{1}$
height of measurement
bark thickness ${ }_{1}$
bark thickness ${ }_{2}$
double bark thickness
sawlog height (merchantable height for sawlogs)

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12 - Exhibit 02
Sample Data Collection Record


| SEGMENT INFORMATION |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Segment Number $\mathrm{XX}$ | Ht. Above Ground XxX.X | Seg. Len. XXX.X | $\begin{aligned} & \hline \text { DIB1 } \\ & \mathrm{xX} . \mathrm{X} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Bark } \\ & \text { Xx.x } \end{aligned}$ | $\begin{aligned} & \hline \text { DIB2 } \\ & \mathrm{xX} . \mathrm{X} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { Bark } \\ & \text { XX.X } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { DOB1 } \\ & \mathrm{XX} . \mathrm{X} \\ & \hline \end{aligned}$ | $\begin{array}{r} \hline \text { DOB2 } \\ \mathrm{XX} . \mathrm{x} \\ \hline \end{array}$ | $\begin{array}{\|r} \text { DBL } \\ \text { Bark } \\ \mathrm{XX} . \mathrm{X} \\ \hline \end{array}$ |
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13 - DATA FIELD DESCRIPTIONS, MEASUREMENT STANDARDS, AND COLLECTION
PROCEDURES.
13.1 - Data Fields. Data elements are defined in FSH 2409.14, Timber
Management Information Systems Handbook. Additional description of
each data field, its coding and where applicable, standards and
procedures to use are illustrated in Exhibits 01-04. See also the
data format, section 12, exhibit 02.
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            13.1 - Exhibit 01
Site and Location Information
```

| Field 1 | Region number/Station number (2-digit numeric). <br> If Stations are doing the work, use the Station reference number of the Station in which the work is located, not the Region number. |
| :---: | :---: |
| Field 2 | Administrative Forest number (2-digit numeric) |
| Field 3 | District number (2-digit numeric) |
| Field 4 | Location ID number (12 characters). |
|  | This is a unique identifier for the project. It must be composed of the Region or Station number, plus a number supplied by the Region or Station to make it unique. The additional number may be a combination of Forest, District, and compartment/stand, and entered as a part of the database loading program with field 4 being the Region or Station portion of the identifier. |
| Field 5 | State (2-digit numeric). |
| Field 6 | County (3-digit numeric). |
| Field 7 | Data source (1-digit numeric). |
|  | Record one of the listed codes. If your data source is not listed in FSH 2409.14, contact the wo data base coordinator to have it included in the list. |
| Field 8 | Coordinate (7-digit numeric). |
|  | If using Universal Transverse Mercator (UTM) Coordinate system, record the UTM Easting in meters and right justify in the field. For <br> latitude/longitude system, record the longitude in degrees (digits $1-3$ of field), minutes (digits 4-5 of field), and seconds (digits 6-7 of field). For state plane coordinate system, record the east-west coordinate in feet, right justified in the field. Be sure to record the state code if state plane coordinates are given. |

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13.1 - Exhibit 01 -- Continued
    Site and Location Information
```

Field $9 \quad$ Y Coordinate (7-digit numeric).
If using UTM Coordinate system, record the UTM
Northing in meters and right justify in the field.
For latitude/longitude system, record the latitude
in degrees (digits $1-3$ of field), minutes (digits
4-5 of field), and seconds (digits 6-7 of field).
For state plane coordinate system, record the
north-south coordinate in feet, right justified in
the field. Be sure to record the state code if
state plane coordinates are given.

Field 10 Universal Transverse Mercator (UTM) Zone (2-digit numeric).

Record the UTM zone found on United States Geological Survey (USGS) topographic maps.

Field $11 \quad \mathrm{X}-\mathrm{Y}$ Coordinate system (1-digit numeric).
Field 12 Date (6-digit numeric).
Record as month/day/year. For example, June 1, 1987 would be recorded as 060187.

Field 14 Site Productivity Class (1-digit numeric).
Record a code for the potential cubic feet mean annual increment.

Field 15 Survey cycle (1-digit numeric).
Field 16 Random number, location (9-digit numeric).
This is a computer-generated random number used in selecting subsets of the data.

Field 17 Compartment number (6-digits numeric). See regional silvicultural examination and prescription handbook or other related regional handbook if available.

Field 18 Stand number (6-digits numeric). See the regional silvicultural examination and prescription handbook or other related regional handbook if available.

Field 19 Note text (up to 256 characters).

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## 13.1 - Exhibit 02

Measurement Information

Field 13 Measurement System (1-character).
Record an $E$ for English an $M$ for metric. Consult the national data base coordinator for standards when using metric units.

Field 105 Crown method (1-digit numeric). Record the code which best describes the technique used.

Code Description
Crown length, compacted (for example, trees with crown on only one side have the crowns mentally shifted to fill in the tree) divided by height of the tree times 100 .

2
Crown length uncompacted (from tip to lowest live limb) divided by the height of the tree times 100 .

Field 116 Diameter measurement technique (1-digit numeric).
Record the code for the technique describing the way the section measurements were made.

## 13.1 - Exhibit 03

Tree Information

| Field 100 | Sample point (4-digit numeric). |
| :---: | :---: |
|  | If the sample is from a design where sample point has no meaning, record a 1; otherwise, record the sample point number. |
| Field 101 | Tree number (4-digit numeric). |
|  | Tree number, combined with the 12-character location identifier and point number, uniquely identifies the tree. The number cannot be duplicated anywhere on the point. |
| Field 102 | Plant Species code (5 characters). See Interim Resource Inventory Glossary. |
|  | Record the species code as shown in FSH 2409.14. For example, lodgepole pine is PICO. |
| Field 103 | Diameter at breast height (dbh) (4-digit numeric) |
|  | Record the dbh in inches and tenths of inches. Measure with a diameter tape before felling the tree. |
| Field 104 | Crown Ratio (3-digit numeric). <br> Record a code corresponding to the percent of live crown. |
| Field 106 | Crown class (1-digit numeric). <br> Record the code for the crown class of the tree. |
| Field 107 | Tree Class (1-digit numeric). <br> Record the code which describes the class of tree. |
| Field 108 | Double bark thickness at dbh (3-digit numeric). |
|  | Record the double bark thickness in inches and tenths of inches. The preferred method of determining this value is to measure the outside bark diameter at the dbh cross section, then measure the inside bark diameter and subtract the two readings to get the bark thickness. When a section cut is not made at dbh, a hatchet may be used, or as a last resort bark gauge. Ensure this measurement is not taken in the bark fissures or where abnormalities occur. |

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13.1 Exhibit - 03 -- Continued
    Tree Information
Field 109 Tree Height(length) (4-digit numeric).
```

Measure the standing tree height using an
appropriate instrument. This measure is ground to
tip. It may be desired for smaller trees to record
this value to the nearest one tenth foot or
centimeter. If the standard field form is used,
specify the decimal.
Field 110 Number of section cuts (2-digit numeric).
Count the number of places where cross-section
information was taken, including the stump and top,
and record that number. Use this field as an edit
check to ensure all of the segment data have been
accounted for.

Field 111 Sequence number (9-digit numeric). Leave blank.
This is a variable which is generated by the data
base software and is useful in finding a particular
record in a file of records.
Field 112 Accumulated height (length) (4-digit numeric).
This field is used to edit the segment data.
Starting from the ground, the segment heights
(lengths) are added together. If this value is
drastically different from the standing tree
height, an error message may occur on database
loading.
Field 113 Record number (6-digit numeric). Leave blank.
This value is generated by the a data base
software.
Field 114
Sawlog height (sawlog length) (4-digit numeric).
Record the height to a user-specified top diameter.
Field 115 Sawlog top diameter inside bark (3-digit numeric).
Measure in inches and tenths of inches at the point
field 114 was measured.
Field 117 Girard form class at 17.3 feet (2-digit numeric).
Record the Girard form class value for top of the
first 16 foot log. Girard form class is
calculated:
[DIB (17.3') x 100] / DBH

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13.1 - Exhibit 03 -- Continued
    Tree Information
Field 118 Girard form class at 33.6 feet (2-digit numeric).
```

Record the Girard form class value for top of the first 32 foot log. Girard form class is calculated:
[DIB (33.6') x 100] / DBH
Field 119 Assumed stump height (3-digit numeric).
In some felled tree studies, the measurements recorded may start at stump height. Actual stump height varies; however, there may be a targeted stump height, such as 1 foot. In this case, record the assumed stump height in feet and tenths of feet.

Field 120 Random number, tree (9-digit numeric). Leave blank.

This number is generated by data base software and is useful for selecting subsets of the data.

Field 121 Elevation (5 digit numeric).
Record the height above sea level referencing topographic maps. This is for the location of the tree, not the elevation of the study as a whole. 3501 feet would be recorded as 3501.

Field 122 Ecological Type (Habitat Type).
Record the code used by the Region or Station to reference the Habitat, Ecoclass, and Vegetation type for the area immediately surrounding the tree in question.

Field 123 Ecological Type (Habitat Type) reference code (2-digit numeric) which describes the Habitat/Ecoclass reference.

Other habitat classifications may be added to the list and assigned a code by the data base coordinator.

Field 124 Aspect (3-digit numeric) azimuth in degrees for the area immediately surrounding the tree in question. Record North as 360 , not zero. Record flat ground as 999.
$\frac{13.1 \text { - Exhibit } 03}{}$ Tree Information

Field $125 \quad$| Slope (3-digit numeric) in percent for the area |
| :--- |
| immediately surrounding the tree in question. |
| Record flat ground as 999. |

Field $126 \quad$ Tree Age (4-digit numeric) at breast height.

Field 127a Height Growth (3-digit numeric).
For selected conifers where annual height growth can be measured, record the height growth for a user assigned number of growing seasons. Measure in feet and tenths of feet.

Field 127b Height Growth Interval (2-digit numeric).
Count the number of growing seasons in the height growth interval.

Field 128 Radial Growth (3-digit numeric).
For selected trees where diameter growth can be measured by ring counts, record the value for the last 10 years in 20th's of an inch. For example, a tree growing 3.5 inches in diameter would have a recorded value of 35 .

Field 129 Basal area per acre (3-digit numeric).
Record the average basal area per acre for the area surrounding the tree being measured. This may be derived as the result of an area wide exam or from installing a prism point with the tree in question as the approximate center. Do not count the tree to be felled.

Field 130 Tree Top Condition (1 character).
Record a code for top damage to the tree (sec. 71 FSH 2409.14).

Field 131 Percent cull (3-digit numeric).
Record the defect amount to the nearest whole percent based on an ocular estimate. (Use a reference to a percent volume breakdown by log for both scribner and cubic measure).

Field 132 Height to Crown, Uncompacted (3-digit numeric).

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13.1 - Exhibit 03 -- Continued
    Tree Information
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Field 133 Height to fork (3-digit numeric).
Record the distance from the ground to the fork in the bole. See FSH 2409.12 for measurement procedures.

Field 134 Site Index (3-digit numeric) for the area surrounding the tree being measured.

Field 135 Site species (5 characters).
The species code for the species on which the site index curves were constructed for calculating the site index recorded in field 135. Use the same species codes as are used for field 102.

Field 136 Site index reference (3-digit numeric).
Record the code for the reference document used to find the site index recorded in field 134. Other site references may be added to the list and assigned a reference code by the data base coordinator.

## 13.1 - Exhibit 04

## Segment Level Information

Field 200 Segment number (2-digit numeric).
Record a unique number for each segment in the tree. See sec. 14.45c.

Field 201 Segment ID (3 characters alpha/numeric).
Record an identifier for each segment. Use this data element for forked trees. Code the main stem as straight segment numbers 1,2,...n. Forks would have a label in field 201 as $F 1$, for the first fork; $F 2$, for the second fork, and so on.

Field 202 Length (3-digit numeric).
Record the length of the segment in feet and tenths of feet.

Field 203 Mid height (4-digit numeric).
Record the height from the ground to the middle of the segment. Measure in feet and tenths of feet.

Field 204 DOB $_{1}$ (4-digit numeric).
Record the first of 2 diameter outside bark measurements, top of the segment, in field 204. Measure in inches and tenths of inches. If only one outside bark measure is made, record in this field. For illustrations on how to treat odd shaped pieces, refer to the Timber Cruising Handbook, FSH 2409.12 and the Cubic Scaling Handbook, FSH 2409.11a.

Field $205 \mathrm{DOB}_{2}$ (4-digit numeric).
Record the second of 2 diameter outside bark measurements, top of the segment. Measure in inches and tenths of inches. The second measurement should be taken at right angles to $\mathrm{DOB}_{1}$.

Field 206 DIB $_{1}$ (4-digit numeric).
Record the first of 2 diameter inside bark measurements, top of the segment. If only one measure is taken, record it in this field. Measure in inches and tenths of inches.

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13.1 - Exhibit 04. -- Continued
```

Segment Level Information
Field $207 \mathrm{DIB}_{2}$ (4-digit numeric). Record the second of 2 diameter inside bark measurements, top of the segment. Measure in inches and tenth of inches. Like Field 205, this second measurement of diameter should be at right angles to the first measurement.

Field 208 Height of measurement (4-digit numeric). Measure in feet and tenths of feet from the ground to the top of the segment being recorded.

Field 209 Bark thickness ${ }_{1}$ Single bark thickness ${ }_{1}$ (2-digit numeric). Record bark thickness at the location of $\mathrm{DIB}_{1}$ and $\mathrm{DOB}_{1}$ measurements. Measure in inches and tenths of inches.

Field 210 Bark thickness ${ }_{2}$ (2-digit numeric). Record bark thickness at the location of $\mathrm{DIB}_{2}$ and $\mathrm{DOB}_{2}$ measurements. Measure in inches and tenths of inches.
Field 211 Double bark thickness (3-digit numeric).
If only one bark measure is available. Record the double bark thickness in this field in inches and tenths of inches.
13.2 - Sampling Procedures. A variety of sampling procedures may be used. See the FSH 2409.12, Timber Cruising Handbook or appropriate references on sampling and select an appropriate procedure.
13.3 - Documentation of Procedures. Prepare a stem profile project synopsis which describes sample design, selection criteria, and field instructions. This synopsis will be an aid to users of the profile data that were not directly involved in the project and will minimize misuse or inappropriate use of data.

14 - FIELD MEASUREMENTS. Select normal trees with a single bole that have no evidence of major current or past damage for the purpose of obtaining data for volume estimators. Basic field procedures for measurements are contained in the Timber Cruising Handbook (FSH 2409.12). Data elements needed for stem profiles are given in previous sections 12-13.1. Additional direction for the field measurements needed for volume estimators is contained in the following sections.
14.1 - Portable Field Recorders. Use field data recorders whenever available for recording the measurements. Arrange for field recorder software procurement and maintenance that meets the standards of this handbook (sec. 11.1 - 14.54).
14.2 - Safety. Taking stem profile measurements in the field and on felled and bucked trees presents potential safety hazards. Ensure that field crews are properly equipped (non-slip boots, hard hats) and are instructed in safe procedures for traveling to the work site and for measuring, felling, and bucking trees. Review appropriate sections of the Health and Safety Code Handbook, FSH 6709.11 prior to initiating the project. Do not sample trees that cannot be felled, bucked, or measured safely.
14.3 - Measurements Made Before Felling. Measure and record the following information while the tree is standing.
14.3 - Table 01

Measurements Made Before Felling the Tree
Diameter Breast Height (dbh)
Total Height (Length) (Ground to tip as described in Field 109)
Sawlog Height (Length) to merchantable sawlog top
Crown Ratio
Height to Crown, Uncompacted
Girard form class 16 or 32 feet
Site information
Slope
Aspect
Elevation
Ecological Type (Habitat Type)
Site index, species, reference table
Location and other ID information
Height to fork
Tree number (painted on the bole)
Basal area/acre
Percent cull
14.4 - Measurements Made After Felling. Measure each tree after it has been felled according to the directions in the following sections.
14.41 - Measurement Points. Take specific measurement at stump height, at a point between stump height and 4.5 feet, at 4.5 feet (breast height), at the form class point (either 16 or 32 feet), at the base of the crown, and at each top diameter outside bark of 5", 4", and 3". As a general rule, take a minimum of measurements at points equal to ten percent of the tree's height. Exhibit 01 illustrates a hypothetical tree with buck points.

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            14.41 - Exhibit 01
Points of Measurement on Typical Tree
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14.42 - Diameter. When there is a choice between using a caliper or actually bucking the tree and recording a cross section area, bucking before measurement should be used since bark thickness can be determined more accurately with a bucked log.
14.42a - Diameter Measurement. When only one measurement is taken, it should be the average diameter as if taken with a diameter tape. If two measurements are taken, one should be the short axis and the other should be perpendicular to the short axis. When both inside and outside diameters are recorded, calculate bark thickness by subtraction.
14.42b - Logs That Cannot Be Moved. If log is pinched, too big, or unsafe to move, use a caliper to measure the diameter. Use a diameter tape to measure the diameter where the end cannot be seen.
14.42c - Stump Diameter. Measure the stump diameter after the tree is felled using the same instrument as used for the other segments. Make this measurement at the first place where a full circumference can be obtained in the case where an under cut makes for an uneven stump surface.
14.43 - Bark Measurements. Make bark measurements with a bark gauge or by subtracting the diameter inside bark from the diameter outside bark. Either method may be used. If bark thickness is to be calculated by subtraction, record the inside and outside bark diameters. The data computation program should calculate the bark thickness.

For felled tree studies, take bark thickness measures at each measurement point. If a bark gauge is used, the measurement can be single bark thickness and so indicated on the field sheet. The double bark thickness is calculated by the data computation program.
14.44 - Height Growth. Count five full growing seasons back from last year's whorl. Cut the tree at the fifth whorl and count the number of rings. Record this value in the height growth interval field on the field form. Measure the distance from the cut to the last year's whorl and record the value in feet and tenths of feet. Be sure to look for past top damage. If the interval is other than 5 years, record the actual number of rings in the height growth interval field; if 10 years, record 10. Additional detail may be found in the Interim Resource Inventory Glossary and in FSH 2409.14, Chapter 70 .
14.45 - Segments.
14.45 a - Cutting the Segments. Cut the bole at each point where diameter measurements are needed. Ensure that field crews follow appropriate safety procedures for bucking. (sec. 14.2 and 14.41).
14.45b - Cuts Which Fall On a Branch Whorl. Since the diameter measurements should not get bigger as the height above ground increases, avoid bucking the tree on a whorl or limb swell to minimize the occurrence of swell points. If a cut must take place on a whorl or limb swell, the axis must not include a knot.
14.45c - Numbering Segments. Starting from the stump, which is segment 1, number each segment by placing the proper number on each segment at the small end.

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14.45d - Segment Length. Measure the distance (height or length) from a diameter measure point to the next diameter measure point with a tape. Use a tape calibrated in feet and tenths of feet, not feet and inches. Start measuring from the ground on the high side and work up the tree. The first segment is from the ground on the high side to top of stump. Most stumps have an uneven surface. Pick the highest surface but ignore stump pulls, barber chairs, and splinters. Match the place of measurement with the same place on the second segment. Continue this practice up to the tip of the tree. Ensure that the sum of the segments equals the height of the tree from ground to tip.
14.45e - Breaks. If the tree breaks when it is felled, visually put the tree back together when measuring segment lengths. The break may or may not be considered a buck point. The choice depends on the particular study. For example, if the study is linked to a particular log scaling rule, buck according to instructions for that rule. See FSH 2409.11 for the Scribner rule or FSH $2409.11 a$ for the cubic rule.

### 14.46 - Irregularities.

14.46a - Missing Pieces. Trees should be rejected from the sample if a piece is missing. To avoid unnecessary measurement, inspect the bole for its full length before taking measurements. If the tree has missing parts, reject the tree.
14.46b - Abnormally Formed Trees. Do not use trees with forks in the main bole, excessive sweep, and crook for stem profile work. Handle forks, crook, and sweep as defect corrections. This includes trees with forks below diameter breast height.
14.47 - Before Leaving the Tree. Edit the data sheet or field data recorder information before leaving the tree and ensure that all required fields are completed. Add the segment lengths and ensure they equal the total height of the tree. Outside bark diameters normally decrease as they progress up the bole; check for errors and confirm any abnormality. Ensure that each set of field instructions mandates these and other field edits deemed to be important for the study or the location.
14.5 - Measurements Made When Trees Are Not Felled. Use optical segmentation procedures for upper stem segment diameters on standing trees in studies where trees are not felled. Measure remaining portions of the tree as described in section 14.3.
14.51 - Types of Equipment. Use the Barr and Stroud Dendrometer, the Spiegel Relaskop with telescope, or laser cruising device to optically determine upper stem diameters. Do not use the Wheeler Pentaprism. For technical instructions on how to use these instruments refer to the manufacturer's user manuals. Use a tripod capable of fine adjustment to support the instrument while measurements are taken.
14.52 - Sighting Points. As with felled tree work, make a minimum of 10 sighting points (measurements points) on the tree. Include stump, diameter breast height, form class point, base of live crown, and at least two sightings between 6" diameter outside bark and the tip. For the remaining sightings, distribute sighting points as evenly along the bole, as possible, but choose areas of opportunity where
visibility is good. See section 14.41 for additional information on measurement points.
14.53 - Instrument Verification. Before taking readings with the instrument, measure the diameter breast height (dbh) with a diameter tape. The first instrument sighting should be at dbh. If the instrument value is significantly different from the tape measurement one of the following will likely be the cause:

1. The bole is excessively elliptical. Reject the tree.
2. The instrument is misadjusted. Adjust the instrument.
3. The instrument is misread. Instruct the instrument person on proper procedure.
4. The distance to the tree may have been misread. Remeasure the distance from instrument to tree.

Make the last measurement a second reading on dbh, providing a way to check if the tripod settled or was tilted while the work was being done. If the second dbh is substantially different from the first, check for error in recording. If none is found, reset the tripod and remeasure the tree.
14.54 - Recording Diameters. Dendrometers, tele-relaskops, and laser instruments may not read diameters directly. Follow instructions for the instrument of choice and the hand held calculator programs available to convert these readings to diameters in the field. Record the field values for diameter in inches and tenths of inches, and the height in feet and tenths of feet. Do not record the instrument values rather than the diameters.

Set up the instruments so the vertical angles are less than 45 degrees. Do not select trees to be measured with the optical instruments if the vertical angle of the instrument exceeds 45 degrees.

Check all field data for completeness, accuracy, and legibility before moving the instrument and before leaving the tree. Incongruities in instrument values discovered later in the office when the diameters are calculated are not correctable since the tree is no longer available for rechecking.

15 - REGION/STATION MODIFICATION OF DATA ELEMENTS OR PROCEDURES.
15.1 - Modification of Collection Format. If modification of the data collection format (sec. 12) is made, verify applicability of the national data loading and edit programs, then modify or replace them as necessary to ensure consistency with data already in the data base.
15.2 - Modification of Data Elements. Do not change definitions of data elements (FSH 2409.14, Ch. 70). Additional descriptive information may be prepared and used in field instructions. If an entirely new element is needed, follow the procedure for non-standard data (sec. 15.3) and request appropriate approval (sec 04.1) for modification of the glossary in FSH 2409.14.

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15.3 - Input of Non-standard Data. When the input of previously
collected, old, or unique data sets into the felled tree data base is
necessary, develop the custom software needed to load the data and
work closely with the designated data base coordinator (sec. 04.13).
Ensure the data is consistent with existing standardized data.
```


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CHAPTER 20 - DATA STORAGE AND MAINTENANCE
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20.2 - Objective. To store tree profile data in a form that is edited, documented, and easily aggregated across geographic and administrative boundaries.
20.3 - Policy. Standardized volume estimator and stem profile data bases shall reside in local data bases used by those preparing volume estimators for timber. Prior to collecting volume estimator and stem profile data, the Region or Station shall design a data base in which to store the data. The design and basic elements of this data base shall conform to national standards and shall be reviewed by the national data base coordinator before collecting field data so that data between Regions and Stations may be aggregated.

21 - DATA BASE DESIGN. Design and prepare a separate data base for each Region and Station. Include the required data elements that are listed in section 12 , exhibit 02.
21.1 - Three Levels of Data. Include three levels of data in each data base:

1. Location data.
2. Tree data.
3. Segment data

Table 01 is a diagram of a three-level data base.
21.1 - Table 01

Diagram of a Three Level Data Base 1/

## LOCATION_TABLE



1/ Establish the relationship between LOCATION and TREE as ONE to MANY and between TREE and SEGMENT as ONE to MANY, since there may be several trees measured for one location, and several segments for a single tree.

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21.11 - Unique Identity. Ensure that each data base and data table
has a unique identity that conforms to the standard format described
in this section. See section 21.1 for a diagram of the
relationships. Name the data base tables as shown in table 01. Use
unique relational data base identification keys for each table as
indicated in table 02. See FSH 2409.14, section 71 for coding of
location ID.
```

                                    21.11 - Table 01
                            Table Names
    1. TBR_VOL_EST_LOCATION_TABLE.
2. TBR_VOL_EST_TREE_TABLE.
3. TBR_VOL_EST_SEGMENT_TABLE.

### 21.11 - Table 02

Unique Relational Data Base Identification Keys

1. TBR_VOL_EST_LOCATION_TABLE, key is:
a. Location ID Number.
2. TBR_VOL_EST_TREE_TABLE key includes:
a. Location ID Number, plus;
b. Sample Point Number, plus;
c. Tree Number.
3. TBR_VOL_EST_SEGMENT_TABLE key includes:
a. Location Number, plus;
b. Sample Point Number, plus;
c. Tree Number, plus;
d. Segment Number, plus;
e. Segment ID, if any.
21.2 - Selected Data Elements. As a minimum, include the required data elements (sec. 12, exhibit 01) in each Region or Station data base for all three tables. See section 13.1, exhibits 01-04 for the field descriptions. Add other optional elements if needed, but ensure that they conform to the Nationwide names of elements as well as their definitions (FSH 2409.14, Chapter 70). Contact the national data base coordinator for assistance (sec 04.13). Contact the Washington Office Director of Timber Management if approval of new elements is necessary (sec. 04.12).

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VERIFICATION OF ESTIMATORS
Use of Standard Statistical Measures
Location and Subsample Differences
Form Class and Merchantable Top Goodness of Fit
Predictor of Standard Tree Results
Comparing Profile and Direct Estimates

Sawn Lumber
Stem Profile Equation Estimators
Direct Lumber Estimators
Board Foot Models
Board Foot/Cubic Foot Ratio Models
Estimation of Losses in Lumber Potential
Veneer
Poles and Pilings
Fiber Products
30.2 - Objective. To describe the derivation and verification of approved stem profile estimators for use by Forest Service mensurationists to ensure accurate and consistent volume estimators used in all timber management functions.
30.3 - Policy. Construct volume estimators in a form that can be used to produce the volume of a tree from ground to tip, as well as to any specified merchantable height or diameter. Product estimates, such as Scribner board feet either shall be a by-product of the total cubic volume or shall be developed from estimates of segment diameter and heights. After developing new volume estimators, implement them in all functions of timber management including cruising, stand exams, and yield tables for forest planning.

31 - ESTIMATOR TYPES AND USES. Consider the types of estimator available for the timber estimation and the potential uses of each within a Region or Station.
31.1 - Stem Profile Equation. For most situations, select a stem profile equation. Consider both the advantages and disadvantages of stem profile methods.

1. Advantages.
a. The equations provide great flexibility in volume calculation.
b. Volume can be calculated to any desired standard of merchantability.
c. Primary products related to log length and diameter can be estimated from diameters calculated without the need for separate product estimators.
d. Stem volumes generated by the integral of a profile equation may be more accurate than those predicted by a direct volume estimator.
2. Disadvantages.
a. Stem profile equations are complex in their derivation and use and are likely to require use of nonlinear least squares (NLLS) regression techniques.
b. Introduction of independent variables other than diameter breast height and height will likely require more work and more computer time than fitting a volume equation.
c. Stem profile equations require a higher degree of training and perceptiveness than fitting volume equations.
31.2 - Direct Volume Estimator. Prepare direct volume estimators only when it is impractical to use stem profile equations. Prepare them in the form of volume tables, alignment charts, or volume equations. Consider both advantages and disadvantages of direct volume estimators.

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1. Advantages.
a. They are less expensive to derive than stem profile equations.
b. Direct volume estimators can be prepared with less skill and effort than stem profile equations.
2. Disadvantages.
a. Direct volume estimators are characteristically loss flexible in use than stem profile equations.
b. Separate direct estimators for different potential products are necessary.
c. When a change in merchantability occurs, the estimator may become obsolete.
31.3 - Product Estimators. Use potential product estimators only to estimate product yield which is very frequently misclassified as volume. Do not use them to estimate volume in the tree, only quantity of product expected to be manufactured from the tree. For example, board feet of sawn lumber is commonly estimated. Consider both advantages and disadvantages of product estimators.
3. Advantages.
a. They are simpler and less expensive to derive when only one product needs to be estimated.
4. Disadvantages.
a. If the assumptions change because of changing technology, the estimator must be redone using the basic tree measurements.
b. Direct potential product estimators have all of the disadvantages of direct volume estimators.
c. The product estimator may become obsolete with changes in manufacturing technology.
d. Assumptions about the manufacturing process are implicit, undocumented, and unknown to the eventual user of the estimator.
31.4 - Selecting the Method Used. Review advantages and disadvantages outlined in sections 31.1-31.3. Choose a method by evaluating the effort required to obtain a stem profile equation and the effort required to prepare a set of merchantable volume and potential product equations that will produce the needed information. Evaluate the expected breadth of application of the results and the duration of application. If only a limited use for a short time is expected, and if having a usable estimator very promptly is important, choose a direct estimator. However, recognize that short cuts taken out of a sense of urgency may lead to inferior analyses that must be done again at a later time. If the volume estimators derived are likely to be used well into the future, use stem profile methodology.

## 32 - PREPARATION OF DATA.

32.1 - Data Evaluation. Check documentation and the adequacy of the data (Ch. 10). Prepare a summary containing the location, dates, and other conditions of measurement supplemented with a detailed report describing the protocol of measurement. Check the species, age classes, and range of heights and diameters of the trees measured to determine suitability of the data base for the proposed volume estimator. Evaluate the number and kinds of abnormal trees (32.22). Prepare simple tables showing frequency of tree data by four or five diameter classes, height classes, and other classes for use in later volume estimation. For a regression analysis, ensure a rectangular (not normal) distribution of tree sizes and sizes and form, if a measure of form is to be included in the estimator. As a first estimate of sample size, use samples of 200 or more trees.
32.2 - Field Data Edit. Edit computer data files for accuracy of transcriptions from field sheets. Check for missing records and delete the tree if key elements of the data are missing. If the initial computer edit program is available, rerun it to verify that the data have been corrected after the preliminary edit.
32.21 - Reasonableness Edits. Use a computer edit program that checks for completeness of records and reasonableness of entries. Ensure it verifies all header entries are complete. Verify that the individual measurements form ascending or descending series and the steps are fairly uniform. If the measurements do not form a series, check them to ensure they are due to bumps, hollows or other stem abnormalities. Establish standards for rejection of pairs of measurements and of entire trees. Use higher standards for measurements of independent variables than for measurements of dependent variables. Also, use higher standards for measurements that may be repeated in a growth study.
32.22 - Abnormal Trees. Evaluate forking or other common abnormalities. Recognize that rejection of trees with abnormalities means that the volume estimator will not correctly estimate trees with similar abnormalities. Trees might not be rejected for species where an abnormality such as forking is common but they should be rejected if the abnormality is uncommon. For example, broken tops may be rare in second-growth stands but nearly universal in old-growth stands. In such cases, reject broken top trees for second growth estimators and not for old growth estimators. Develop appropriate standards for rejection of abnormal trees in all projects.

Estimate volumes of abnormal trees separately if a large enough sample is measured. If their volumes are consistently and significantly different from other trees of the same species and the abnormality is prevalent in significant parts of the area of interest, prepare separate volume estimators.
32.23 - Graphic Edits. Use individual tree plots to quickly identify pairs of measurements that appear to be unusual, and to identify trees with broken tops or other abnormalities. Use this kind of editing when it is more effective than defining limits and flagging the measurements exceeding them. Identify all unusual and unforeseen conditions that are indicated.

Use scatter diagrams for groups of trees to suggest the average shape for the group, and flag the presence of individual trees that deviate

## TIMBER VOLUME ESTIMATOR HANDBOOK

widely from this average. Use changes in average shapes among groups of trees to suggest functions to be tested in developing the volume estimator. Use the simple diagrams to identify transformations of basic measurements that have straight line relations with other measured variables. Use $X-Y$ scatter diagrams both to edit data and as an early step in data analysis.
32.3 - Preparing Data for Analysis. Prepare data for analysis at the time edits are made.
32.31 - Field Computation. Convert dendrometer and relaskop data to diameters, heights, and distances in the field, and correct any problems before leaving the instrument setup point (sec. 14.47).
32.32 - Average Diameters. If more than a single diameter is recorded at any point other than breast height, average them and save at least two decimals. These may help compensate for
out-of-roundness at different heights above the ground. Assess the degree of out-of roundness before deciding the usefulness and accuracy of using two breast height diameters and then specify how to record. Identify the reasons for saving two breast height diameters, such as standing trees measured with diameter tapes and felled trees measured with calipers.
32.33 - Bark Thickness. If for any reason the recorded bark thickness is not the sum of two measurements, clearly indicate this in the header information. Use bark thickness to generate inside-bark or outside-bark diameters from the measured diameter.
32.34 - Relative Diameter. Determine relative diameters by dividing the inside bark diameters recorded at each measurement point by an inside bark basal diameter, most commonly diameter breast height (dbh). If possible, do not use outside bark measurement because bark ratio is not constant at all measurement points along the bole. Use relative diameter measured outside bark only when it is not practical to get inside bark measurements.
32.35 - Relative Height. Express relative height as the distance from tip of a tree to height of measurement point divided by either total height or by total height minus breast height. Either makes zero relative height coincide with zero relative diameter, and the latter makes the 1.0 points coincide. Use this convention for relative height to simplify equations in the volume estimator.
32.4 - Working Files. Maintain at least two kinds of working files. In the first file known as the profile working file, include the record of tree data (heights, diameters, and bark-thicknesses at various levels in the tree) (sec 32.41). In the second, include the record of section and tree volumes (sec 32.42).
32.41 - Profile Working Files. In profile working files, include one line of data for each height of measurement (elevation). Include about 10 spaces for tree and plot identification (including tree species) in each line. Follow this with tree dbh and tree height as a minimum and optionally, additional tree measurement data. Put any plot or location data that is to be used in the analysis next in the sequence. Record the elevation number, starting with zero for ground level to ensure that the elevation number of the top of each section coincides with the section number; "one" usually is the stump, making breast height elevation usually "two" or "three."

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Follow the elevation numbers with the cumulative heights in feet, the diameters outside bark, the double bark thickness and the diameter inside bark. Lastly enter the relative heights and diameters (both outside and inside bark). Use a separate column with indicators of first and last measurement of each tree when there are variable numbers of lines of data for trees.
32.42 - Section Volume Files. Calculate section volumes using the profile working file. Use procedures that avoid bias such as the following:

1. Since few trees are cylindrical, conduct a simple study to find a multiplying factor such as 1.3 or 1.5 that removes the bias of the textbook assumptions. Often stem profiles from the top of the stump to an inflection point 15 percent to 25 percent of the height of the tree are concave. Calculation of the volume of stem segments in this zone as frustra of cones or paraboloids (FSH 2409.11a, sec. 11.1) can give biased estimates. Keep the measurements close together to keep this bias at one percent or less.
2. Tree tips are usually assumed to be cones but are often somewhat more convex. Unless a more accurate multiplier has been determined in the field, assign volumes halfway between a cone (K/3) and a paraboloid (K/2) to reduce potential bias for tips of trees. Add a column of calculated volumes for each segment to the profile working file.
32.43 - Tree Volume Files. Prepare a separate file of tree volumes. Enter tree information similar to that in the profile working files, as well as any other information that may be used as an independent variable. In addition to stem volumes from stump to various merchantable tops, include stump volume and tip volumes above the various merchantable tops.
32.44 - Temporary Files. If temporary files are created in generating the working files, label these clearly and delete them when no longer needed. Consider the usefulness of preserving intermediate results as part of the working files before the temporary files are deleted. Store the temporary files with the data files, but in a manner that precludes confusing them with the permanent information.

33 - STEM PROFILE EQUATIONS. Stem profile equations are a mathematical expression relating diameter of a tree bole to height above ground at any point between ground level and the tree's tip. Recognize that the terms taper equation and taper function are often used interchangeably in scientific literature. Use of the term profile equation is recommended. Estimate parameters of the equation by regression. Use diameter, or a transformation of diameter, as the regression dependent variable. Use height to the point of prediction, or a transformation of such height, as the basic independent variable. Include other independent variables, if needed, to describe variation in tree form due to site conditions or stand density. Ordinarily, make the dependent regression variable the ratio of squared stem diameter to squared dbh, since scaling of diameters is necessary to accommodate trees of all sizes, and makes residual variance about regression homogeneous.

After estimating equation parameters, rearrange stem profile equations algebraically to predict squared stem diameter as a

## TIMBER VOLUME ESTIMATOR HANDBOOK

function of squared $d b h$ and relative height. This facilitates computing stem volume by integrating the stem profile equation.
33.1 - General Form of a Stem Profile Model. Use the general form of a stem profile model which is:

$$
\mathrm{d}=\mathrm{f}[\mathrm{D}, \mathrm{~h}, \mathrm{H}]
$$

where $D$ is tree dbh, $H$ is total tree height, and $d$ is the desired bole diameter at a distance $h$ from the ground. Stated simply, bole diameter is a function of the dbh and relative height of that diameter. Use relative diameters and heights when fitting equations to reduce the effect of tree size on bole diameter predictions.
33.2 - Desirable Characteristics. From among the different model forms available (sec. 33.32), choose the best model based on desirable properties. The model should be flexible, accurate, and able to produce results consistent with expectations of tree shape.
33.21 - Model Flexibility. Plan for maximizing flexibility of the model to fit the wide variety of geometric shapes a tree bole may approximate. For example, the lower bole may be concave and the upper portion a convex paraboloid. Develop a flexible model that conforms to the varied bole shapes which may be concave near the base, convex in the middle, and conical or paraboloid at the top.
33.22 - Model Accuracy. Strive for a model that produces accurate diameter estimates all along the bole from the ground to the tip. Since models are less accurate at the base of the tree due to butt flair and at the tip due to bole irregularities, aim for acceptable accuracy from 2 percent of the total height to about 90 percent of the total height. Do not expend major effort in trying to fit the stump and top of the tree, since improvement in diameter estimates in these portions of the tree is not cost effective.
33.23 - Expectations. Regardless of equation complexity, ensure that the predictions yield results consistent with user expectations about tree profiles. Solve the profile function directly for the following three variables: 1) diameter (d) at a measured height (h); 2) height (h) to a point of known bole diameter (d); and 3) bole cubic-foot volume calculated from the stump to the point of bole
merchantability. Use calculus to integrate the equation over the desired height limits and ensure that the model meets the following expectations:

1. DBH. At a height of 4.5 feet, the predicted outside bark bole diameter is expected to equal the measured dbh. Exact results at dbh, help maintain the model credibility with the user. If the profile equation does not naturally yield this result, condition the basic model to equal $D$ when $h=4.5$ or:

$$
f(x)=D \text { when } h=4.5 ; \quad X=(H-4.5) / H .
$$

2. Tip of the tree. At the tip of the tree the diameter is expected to be zero. When $h=H$ the profile equation should predict a zero diameter or:

$$
f(x)=0 \text { when } h=H ; X=(H-H) / H .
$$

If the model does not produce the result, condition the base model to yield this result prior to fitting it to the data.
3. Smaller diameters at greater heights. The profile equation is expected to predict successively smaller diameters as the height increases up the bole. Although irregularities on the bole are common, do not try to predict convolutions on the bole, rather ensure the model predicts a smooth trend of decreasing diameters from the base to the tip.
33.24 - Other Considerations. In addition to equation flexibility, accuracy, and prediction expectation, consider the following items before deciding on a particular model form to use:

1. Use $Y=(d / D)^{2}$ or $Y=(d / D)$. When deriving a stem profile model make an early choice of the dependent variable, $Y=(d / D)$ or $Y=(d / D)^{2}$.

When possible, choose (d/D) ${ }^{2}$ because cubic volume is a function of the mean cross-sectional area of the bole which, in turn, is directly related to (d/D) ${ }^{2}$. Thus, obtain bole volume by directly integrating the function of $(d / D)^{2}$.

If a profile equation using $Y=(d / D)$ is chosen, square the entire function prior to integration to predict cubic volume. The squaring process may become a complex of algebraic manipulations, therefore, avoid this process when possible.
2. Integrable in closed form. If possible, choose a model that is integrable in closed form. If the equation cannot be integrated directly, then use a numerical integration technique to calculate volume. Integration requires many calculations per tree, which causes a higher computer cost. Do not, however, avoid profile functions because of this complexity; simply recognize the difficulty and possible increased expense.
3. Easily solved merchantable height. If possible, choose equations that can be algebraically solved for h. Profile function may be used to estimate an average merchantable height (h) given a known, dbh, tree height, and top diameter. Equations that must use numerical methods to solve for $h$ involve the same calculation and cost problems as discussed in item 2 above.
33.3 - Profile Model Examples. Choose profile models from the following examples of models that range from simple to complex. Generally, the simplest models are not suitable for most applications since they are not accurate over the full length of the bole. The more complex models often give like results over much of the bole length, and usually differ only at the ends of the bole.
33.31 - Variables. Variables unique to a particular model are defined with that model as it appears. The variables used for the models and equations are presented in Table 01.

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33.31 - Table 01

Notation for Variables Used
$D=d b h=$ Tree diameter breast high (4.5 feet).
H = Tree height in feet.
d = A diameter being estimated at some point $h$ on the bole.
dob = diameter outside bark.
dib = diameter inside bark.
$h=$ Height above the ground to $d$.
L = Lower limit of bole merchantability, feet above the ground.
U = Upper limit of bole merchantability, feet above the ground.
$\mathrm{K}=0.005454$ - converts diameter squared in square inches to square feet. b $=$ Coefficients estimated from a specific data set.
33.32 - Models. Select from among the 10 standard tree profile models presented in this section. They range from models of very simple form and poor accuracy to complex and accurate models. Exhibit 01 assigns numerical evaluations to some desirable model characteristics while designating others with a simple yes or no answer. Models which are rated as simple lack accuracy, while those that are rated accurate are the most complex.

No one model is best for all circumstances. Exhibits 02-12 illustrate the 10 models. Models in exhibits 08-12 usually give accurate results for a wide variety of species and geographic locations. When deriving stem profile equations for a particular species or area, test a number of model forms for suitability.

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### 33.32 - Exhibit 01

A Tabular Evaluation of Models

| Model | $\begin{aligned} & \text { Simplic- } \\ & \text { ity } \end{aligned}$ | $\mathrm{h}=\mathrm{f}(\mathrm{d})$ | Closed <br> for V | $\begin{aligned} & \text { Accu- } \\ & \text { racy } \end{aligned}$ | Flex- <br> ibility | Reliability | Upper bole <br> d needed | Vol sets |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ormerod | 5* | Yes | Yes | 1 | 1 | 1 | No | Yes |
| ```Behre hyperbola``` | 5 | Yes | Yes | 1 | 1 | 1 | No | Yes |
| Kozak et al. | 4 | Yes | Yes | 2 | 2 | 2 | No | No |
| Amidon | 3 | No | Yes | 3 | 3 | 3 | No | No |
| Demaerschalk compatible | 3 | Yes | Yes | 2 | 2 | 2 | No | Yes |
| Biging | 2 | Yes | Yes |  | -not rat | d--- | No | No |
| Demaerschalk and Kozak | 1 | No | No | 4 | 5 | 4 | Yes | No |
| Bruce et al.** | 3 | No | Yes | 4 | 5 | 5 | No | No |
| Schlaegel <br> form class | 2 | No | Yes | 5 | 5 | 5 | Yes | No |
| $\begin{aligned} & \text { Max - } \\ & \text { Burkhart** } \end{aligned}$ | 2 | Yes | Yes | 5 | 5 | 5 | No | No |

*Each equation has been evaluated for a number of qualities. The numeric rating denotes the degree to which the quality is met with 5 being the highest level of attainment. Yes/no denotes whether or not the function possesses the quality.
**Both equations are highly regarded in application. Numerous applications are referenced in the literature.

The Ormerod Model

1. Simple model. The simplest form of the Ormerod model is the generalized paraboloid.
$d / D=X^{b}$
where
$X=(H-h) /(H-4.5) . \quad$ If $b<1.0$, the bole shape will be
parabolic. If $b>1.0$, the shape will be neiloidal.

Solving for $h$ given $d:$
$h=H-\left[(H-4.5)(d / D)^{1 / b}\right]$.

Ormerod volume function:

$$
V=\frac{K D^{2}(H-4.5)}{Z}\left[\left(\frac{(H-L)}{H-4.5}\right)^{z}-\left(\frac{(H-U)}{H-4.5}\right)^{Z}\right]
$$

where:

$$
Z=2 b+1
$$

2. Expanded Ormerod model. The simple form of the Ormerod model assumes one bole shape extending from the butt to the tip and is a poor profile model. Tree boles change shape from the butt to the tip. Hence, the model may be expanded as a step function, with separate equations fitted for each bole segment. This form of the model offers increased flexibility; however, a diameter ( $D_{i}$ ) and height $\left(H_{i}\right)$ must be measured on each segment, making it's use very expensive.

$$
d_{i}=\left(D_{i}-C_{i}\right) X_{i}^{b_{i}}+C_{i}
$$

where:
$d_{i}=$ estimated diameter at height on segment $i$
$D_{i}=$ measured diameter at height $k$ on segment $i$
$H_{i}=$ height to the top of segment $i$
$C_{i}=$ diameter intercept of segment $i$
$X_{i}=\frac{\left(H_{i}-h\right)}{\left(H_{i}-k\right)}$

## The Behre Hyperbola

1. Profile model: A simple model useful where less accurate estimates are needed.

$$
\frac{d}{B}=\frac{X}{b_{0}+b_{1} X}
$$

where:
$B=A$ basal diameter in inches; may be d.b.h
$X=(H-h) / H$
$H=$ The bole length in feet measured from the height of the basal diameter to the tip of the tree
$h=H e i g h t ~ a b o v e ~ t h e ~ b a s a l ~ d i a m e t e r ~ t o ~ d ~$ and a restriction that $b_{0}+b_{1}=1$
2. Solving for $h$ given $d$ :

$$
h=H\left[1-\left(b_{0} d\right) /\left(B-b_{1} d\right)\right]
$$

3. Volume function:

$$
\begin{aligned}
V & =\frac{K B^{2} H}{b_{1}^{3}}\left\{b_{1}\left(X_{b}-X_{t}\right)+2 b_{0} \operatorname{Ln}\left[\frac{b_{0}+b_{1} X_{t}}{b_{0}+b_{1} X_{b}}\right]\right. \\
& \left.+b_{0}^{2}\left[\frac{1}{b_{0}+b_{1} X_{t}}-\frac{1}{b_{0}+b_{1} X_{b}}\right]\right\}
\end{aligned}
$$

where:
$X_{b}=(H-L) / H$
$X_{t}=(H-U) / H$
and
Volume from the stump to the height of $B$
is computed as a simple geometric solid
4. Considerations.
a. May be applicable in the Northwest.
b. Contains two coefficients, but uses essentially a single parameter, since $\mathrm{b}_{0}=1-\mathrm{b}_{1}$, thus, $\mathrm{b}_{1}=(\mathrm{d}-$ BX)/[d(1 - X)].
c. Fit the equation by finding $b_{1}$ for each pair of $d$ and $X$ and the average of the $\mathrm{b}^{\prime}$ 's to get a single $\mathrm{b}_{1}$. As $\mathrm{b}_{1}$ increases from 0 to 1 , the hyperbola represents trees of higher form class.
d. For use, good estimates of standing tree form class are required.

### 33.32 - Exhibit 04

The Kozak, et al. Model

1. Profile model. The taper model proposed by Kozak, et al., is a parabolic function which assumes the entire bole is shaped as a parabola. Butt swell is assumed to be a slight and unimportant component of tree volume. The model is:

$$
\frac{d^{2}}{D^{2}}=b_{1} \frac{(h-H)}{H}+b_{2} \frac{\left(h^{2}-H^{2}\right)}{H^{2}}
$$

2. Solving for $h$ given $d$. The height $h$ to a given top diameter $d$ if $D$ and $H$ are known is given by:

$$
h=\left\{-b_{1}-\left[b_{1}^{2}-4 b_{2}\left(b_{0}-\frac{d^{2}}{D^{2}}\right)\right]^{1 / 2}\right\}\left(H / 2 b_{2}\right)
$$

where:

$$
b_{0}=-b_{1}-b_{2}
$$

3. Kozak, et al. Volume function:

$$
V=K D^{2}\left[b_{0}(U-L) \frac{b_{1}\left(U^{2}-L^{2}\right)}{2 H}+b_{2} \frac{\left(U^{3}-L^{3}\right)}{3 H^{2}}\right]
$$

where:

$$
b_{0}=-b_{1}-b_{2}
$$

4. Considerations.
a. It may be useful where tree form closely approximates a parabola or where the target population consists of small diameter trees having minimum butt swell. In these situations, more complex models may not produce a better fit.
b. Do not use if precise volume estimates of individual trees are needed.

The Amidon Model

1. Profile Model:

$$
d=b_{1} \frac{D(H-h)}{(H-4.5)}+b_{2} \frac{\left(H^{2}-h^{2}\right)(h-4.5)}{H^{2}}
$$

2. Solving for $h$ given d. Use an interactive approximation procedure to predict a merchantable height given a merchantable top diameter. No analytical solution exists.
a. The Newton-Raphson iteration. Use the simple iterative approximation, the Newton-Raphson iteration. Set the profile equation equal to the diameter to be solved and find the respective height by iteratively guessing the solution. Use a five step process:
(1) Make an initial guess at the supposed height $h_{0}$;
(2) Solve the profile equation for $h_{0}$; compute $f\left(h_{0}\right)$;
(3) Find the first derivative in $h$ of the profile equation and solve for $h_{0}$; compute $f^{\prime}\left(h_{0}\right)$;
(4) Compute the new height $h_{1}=h_{0}-f\left(h_{0}\right) / f^{\prime}\left(h_{0}\right)$;
(5) Set $h_{0}=h_{1}$ and repeat steps 1 through 4 until the correction factor $C=-f\left(h_{0}\right) / f^{\prime}\left(h_{0}\right)$ ceases to change by some previously specified value, say $0.002,0.1,0.5$ or maybe 1.0 foot.
b. Derivative in $h$ of Amidon Model:

$$
f^{\prime}(h)=b_{2} \frac{\left(H^{2}-3 h^{2}+9 h\right)}{H^{2}}-\frac{b_{1} D}{(H-4.5)}
$$

3. Amidon Volume Function:

$$
\begin{aligned}
V & =K\left[Z_{0}^{2}(U-L)+Z_{0} Z_{1}\left(U^{2}-L^{2}\right)+\left(Z_{1}^{2}+2 Z_{0} Z_{1}\right)\left(U^{3}-L^{3}\right) / 3\right. \\
& +\left(Z_{0} Z_{3}+Z_{1} Z_{2}\right)\left(U^{4}-L^{4}\right) / 2+\left(Z_{2}^{2}+2 Z_{1} Z_{3}\right)\left(U^{5}-L^{5}\right) / 5 \\
& \left.+\left(Z_{2} Z_{3}\right)\left(U^{6}-L^{6}\right) / 3+Z_{3}^{2}\left(U^{7}-L^{7}\right) / 7\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
& Z_{0}=\frac{b_{1} D H}{2(H-4 \cdot 5)}-\frac{4 \cdot 5 b_{2}}{2} \\
& Z_{1}=\frac{b_{2}}{2}-\frac{b_{1} D}{2(H-4 \cdot 5)} \\
& Z_{2}=\frac{4 \cdot 5 b_{2}}{2 H^{2}} \\
& Z_{3}=-\frac{b_{2}}{2 H^{2}}
\end{aligned}
$$

4. Considerations.
a. For five California conifers this model accurately predicted bole diameters of and ranked first over five other models of proven accuracy including the Max and Burkhart, Bruce, et al., Kozak, et al., and Demaerschalk exhibited in section 33.32 .
b. Fit the model inside bark for given $H$ and D outside the bark. So when $h=4.5$, the predicted diameter is $d=$ $\mathrm{b}_{1} \mathrm{D}$. Thus, the double bark thickness at breast height is $B=D\left(1-b_{1}\right)$.

## The Demaerschalk Model

1. Profile Model:

$$
\frac{d^{2}}{D^{2}}=10^{2 b_{0}}\left[D^{\left(2 b_{1}-2\right)}\right](H-h)^{2 b_{2}} H^{2 b_{3}}
$$

2. Solving for $h$ given $d$. The height $h$ to a given top diameter $d$ if $D$ and $H$ are known is given by:

$$
h=H-\left[10^{-b_{0}} d D^{-b_{1}} H^{-b_{3}}\right]^{1 / b_{2}}
$$

3. Demaerschalk volume function:

$$
V=K(10){ }^{2 b_{0}} D^{2 b_{1}} H^{2 b_{3}}\left(X_{1}^{Z}-X_{2}^{z}\right)
$$

where:

$$
\begin{aligned}
Z & =\left(2 b_{2}+1\right) \\
X_{1} & =H-L \\
X_{2} & =H-U
\end{aligned}
$$

If a tree volume equation of the form:
$\ln (\mathrm{V})=\mathrm{a}+\mathrm{b}(\ln (\mathrm{D}))+\mathrm{c}(\ln (\mathrm{H}))$
already exists, then convert it into a logarithmic taper equation of the form:
$\ln (\mathrm{d})=\mathrm{b}_{0}+\mathrm{b}_{1}(\ln (\mathrm{D}))+\mathrm{b}_{2}[\ln (\mathrm{H}-\mathrm{h})]+\mathrm{b}_{3}(\ln (\mathrm{H}))$,
where:
a, b, and c are the volume equation coefficients and
$\mathrm{b}_{0}=\left(10^{\mathrm{a}} \mathrm{pc} / \mathrm{K}\right)^{1 / 2} ;$
$\mathrm{b}_{1}=\mathrm{b} / 2$;
$\mathrm{b}_{2}=(\mathrm{pc}-1) / 2$;
$\mathrm{b}_{3}=(1-\mathrm{p}) \mathrm{c} / 2$;
$\mathrm{p}=\mathrm{a}$ "free" parameter which provides the compatibility between the volume equation and the profile equation.

The Demaerschalk Model
Or derive and integrate an alternative profile function of the form:

$$
\ln (\mathrm{d})=\mathrm{b}_{0}+\mathrm{b}_{1}(\ln (\mathrm{D}))+\mathrm{b}_{2}[\ln (\mathrm{H}-\mathrm{h})]+\mathrm{b}_{3}(\ln (\mathrm{H})),
$$

providing a compatible volume equation:

$$
\ln (V)=a+b \ln (D)+c \ln (H)
$$

where:

$$
\begin{aligned}
& a=\ln \left[\frac{K(10)^{2 b_{0}}}{2 b_{2}+1}\right] \\
& b=2 b_{1} \\
& c=2 b_{2}+2 b_{3}+1
\end{aligned}
$$

## 4. Considerations.

a. Use this model in situations where intermediate levels of accuracy and complexity meet the needs. For example, if the intent is to predict either total cubic foot volume or cubic foot volume to a pulpwood top, this is acceptable.
b. The profile equation is usually not accurate over the entire bole length for predicting bole diameter.
c. This model integrates mathematically to give the exact same volume estimate predicted from an already existing total stem volume equation, and demonstrates the concept of compatible profile and volume equations.

## The Biging Model

1. Profile model. The model is based on the integral form of the Chapman-Richards function. Biging's model is:

$$
d=D\left\{b_{1}+b_{2} \ln \left[1-\lambda(h / H)^{1 / 3}\right]\right\}
$$

where:

$$
\lambda=\left[1-\exp \left(-b_{1} / b_{2}\right)\right]
$$

2. Solving for $h$ given $d$. To predict a merchantable height $h$ to a given top diameter $d$ :

$$
h=H\left\{\left[1-\exp \left(\frac{\left(d-b_{1} D\right)}{b_{2} D}\right)\right] / \lambda\right\}^{3}
$$

3. Biging Volume Function: Use the following formula:

$$
\begin{aligned}
V & =K \int_{L}^{U} d^{2}(h) d h \\
& =K_{1} H(U 1-L 1) \\
& +\left.K_{2} H\left[-\frac{3}{\lambda^{3}}\right]\left[q \ln (q)-q-q^{2}-\ln (q)+\frac{q^{2}}{2}+\frac{q^{3}}{3} \ln (q)-\frac{q^{3}}{9}\right]\right|_{L 1} ^{U 1} \\
& +K_{3} H\left[-\frac{3}{\lambda^{3}}\right]\left[\frac{q^{3} \ln ^{2}(q)}{3}-q^{2} \ln ^{2}(q)+q \ln n^{2}(q)-\frac{2}{9} q^{3} \ln (q)\right. \\
& \left.+q^{2} \ln (q)-2 q \ln (q)+\frac{2}{27} q^{3}-\frac{q^{2}}{2}+2 q\right]\left.\right|_{L 2} ^{U 2}
\end{aligned}
$$

where:

$$
\begin{aligned}
L & =\text { height to a stem base point; could be stump } \\
U & =\text { height to a top point in the upper stem } \\
h & =\text { height to a point between } L \text { and } U \\
\lambda & =\left[1-\exp \left(-b_{1} / b_{2}\right)\right] \\
q & =\left[1-\lambda(h / H)^{1 / 3}\right] \\
L 1 & =L / H \\
L 2 & =\left[1-\lambda(L / H)^{1 / 3}\right] \\
U 1 & =U / H \\
U 2 & =\left[1-\lambda(U / H)^{1 / 3}\right] \\
K 1 & =K b_{1}^{2} D^{2} \\
K 2 & =K\left(2 b_{1} b_{2} D^{2}\right) \\
K 3 & =K b_{2}^{2} D^{2}
\end{aligned}
$$

### 33.32 - Exhibit 07 continued

The Biging Model
4. Considerations.
a. Use this model in situations where need for intermediate accuracy exists.
b. The Biging equation is complex.
c. Fit this model to bole diameters inside the bark even though dbh is measured outside the bark. Thus, when $h=H$, then $d=0$, and when $h=0$, then $d=b_{1}$ (dbh). Interpret coefficient $\mathrm{b}_{1}$ to be the ratio of dib at the base of the tree to dbh. The equation may be fitted to bole diameters outside the bark, and if so, interpret the coefficient $b_{1}$ to be the ratio of bole dob at the stem base to dbh.
d. This model has applicability for ponderosa pine, Douglas- fir, white fir, red fir, sugar pine, and incense cedar. It equaled the Max-Burkhart model for prediction accuracy over the range of the collected data. The Biging model may be a good alternative to the Max-Burkhart model for some applications.

### 33.32 - Exhibit 08

The Demaerschalk and Kozak Dual-Equation Model

1. The profile model. Use two models for different portions of the tree. Since a single profile equation does not adequately describe bole profile from ground to tip, use one model to describe upper bole profile and another to describe lower bole profile.

These profile equations predict tree profile inside bark since this is the usual variable of interest when predicting tree volume. Since dbh inside bark (DIB) is not measured directly, predict it from the data using the model:

$$
\mathrm{DIB}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{DBH}+\mathrm{b}_{2}(\mathrm{DBH})^{2} .
$$

Likewise, predict the diameter inside bark at the inflection point (DI) from:
$D I=C_{0}+C_{1} D I B+C_{2}(D I B)^{2}$.
a. The model for the bole from the tree tip down to the inflection point is:

$$
\frac{d}{D I}=\left[\left[\frac{h / H}{R}\right]^{b_{1}} b_{2}^{\left(1-\frac{h / H}{R}\right)}\right]
$$

where:

$$
\begin{aligned}
h= & \text { the distance from the tree tip to d } \\
D I= & \text { diameter inside bark at the inflection point } \\
R= & \text { distance of the inflection point from the } \\
& \text { tip relative to } H
\end{aligned}
$$

```
33.32 - Exhibit 08 -- continued
```

The Demaerschalk and Kozak Dual-Equation Model
b. The bottom model from the inflection point to the ground is:

$$
\frac{d}{D I}=\left[b_{3}-\left(b_{3}-1\right)\left[\frac{1-h / H}{I}\right]\right.
$$

where:

$$
\begin{aligned}
I= & 1-R, \text { the relative height of the } \\
& \text { inflection point from ground level }
\end{aligned}
$$

$$
b_{3}=\frac{\frac{D I B}{D I}-\left[\frac{1-\frac{H-4.5}{H}}{I}\right]^{b_{4}}}{1-\left[\frac{1-\frac{H-4.5}{H}}{I}\right]^{b_{4}}}
$$

2. Solving for $h$ given $d, D$, and $H$.
a. For the bottom model.

$$
h=H\left[1-I\left[\frac{b_{3}-d / D I}{b_{3}-1}\right]^{1 / b_{4}}\right]
$$

where:

$$
h=\text { the distance from the top }
$$

b. For the top model. Solve for $h$ using an iterative procedure, such as Newton-Raphson, since no analytical solution exists. The first derivative of the top model with respect to $h$ is:

$$
F^{\prime}\left[f\left[\frac{d}{D I}\right]\right]=\left[\frac{b_{2}}{H R^{b_{1}}}\right]\left[b_{1} h^{\left(b_{1}-1\right)} b_{2}^{-\left(\frac{h}{R H}\right)_{-}}\left[\frac{h^{b_{1}}}{H R}\right] b_{2}^{-\left(\frac{h}{R H}\right)} \ln \left(b_{2}\right)\right]
$$

TIMBER VOLUME ESTIMATOR HANDBOOK

### 33.32 - Exhibit 08 -- continued

The Demaerschalk and Kozak Dual-Equation Model
3. Computing cubic volume. Obtain cubic volumes for the bottom model by squaring the model and integrating over desired limits. Obtain the top model volume by finding the top and bottom diameters of small (0.5 foot) segments using the derived taper equation, computing the segment volumes using Smalian's equation, then summing the segment volumes. Add the top volume and bottom volume to get tree volume.

To make volume computations uniform for the whole tree, solve both models by summing small segments. Therefore, no integral form of the bottom is presented.
4. Considerations.
a. The models join at the inflection point, the point on the bole where the shape of the bole changes from the neiloidal base to the parabolic top. For each species group the inflection point differs. But, for all species it ranges from 20 to 25 percent of the tree height measured from ground level.
b. Condition the models so the predicted diameter is 0 at the tree tip, equals dbh at 4.5 feet, both are smooth and equal DI at the inflection point.
c. Since the bole inflection point cannot be visually determined, determine it for each species group prior to fitting the equations. That point is constant for all trees in that group. Determine $R$ before fitting the equations.
d. The bottom model has two coefficients and two restrictive conditions; therefore, the bottom equation is unique for each tree. Coefficient $b_{4}$ is determined for each tree by iteration and $b_{3}$ is set equal to a function of $b_{4}$. Consider this to be a drawback in using this prediction system.

TIMBER VOLUME ESTIMATOR HANDBOOK
33.32 - Exhibit 09

The Bruce et al. Red Alder Model

1. Profile Model:

$$
\begin{aligned}
\frac{d^{2}}{D^{2}} & =b_{1}\left(X^{3 / 2}\right)\left(10^{-1}\right) \\
& +b_{2}\left(X^{3 / 2}-X^{3}\right) D\left(10^{-2}\right) \\
& +b_{3}\left(X^{3 / 2}-X^{3}\right) H\left(10^{-3}\right) \\
& +b_{4}\left(X^{3 / 2}-X^{32}\right) D H\left(10^{-5}\right) \\
& +b_{5}\left(X^{3 / 2}-X^{32}\right) H^{1 / 2}\left(10^{-3}\right) \\
& +b_{6}\left(X^{3 / 2}-X^{40}\right) H^{2}\left(10^{-6}\right)
\end{aligned}
$$

where:

$$
X=\left[\frac{(H-h)}{(H-4.5)}\right]
$$

TIMBER VOLUME ESTIMATOR HANDBOOK
33.32 - Exhibit 09 -- continued

The Bruce et al. Red Alder Model
2. Solving for $h$ given $d$. Since the profile model is a complex polynomial, a unique solution to estimate $h$ given $d, D$, and $H$ does not exist. Use an alternative approximation procedure (as for Amidon, 32.32, ex. 05) which, necessitates finding the first derivative of the profile function with respect to $h$ :

$$
\begin{aligned}
f^{\prime}(h) & =-b_{1}\left[\frac{3 M^{1 / 2}}{2 N^{3 / 2}}\right]\left(10^{-1}\right)+b_{2}\left[\frac{3 M^{2}}{N^{3}}-\frac{3 M^{1 / 2}}{2 N^{3 / 2}}\right] D\left(10^{-2}\right) \\
& +b_{3}\left[\frac{3 M^{2}}{N^{3}}-\frac{3 M^{1 / 2}}{2 N^{3 / 2}}\right] H\left(10^{-3}\right) \\
& +b_{4}\left[\frac{32 M^{31}}{N^{32}}-\frac{3 M^{1 / 2}}{2 N^{3 / 2}}\right] D H\left(10^{-5}\right) \\
& +b_{5}\left[\frac{32 M^{31}}{N^{32}}-\frac{7 M^{5 / 2}}{2 N^{7 / 2}}\right] H^{1 / 2}\left(10^{-3}\right) \\
& +b_{6}\left[\frac{40 M^{39}}{N^{40}}-\frac{3 M^{1 / 2}}{2 N^{3 / 2}}\right] H^{2}\left(10^{-6}\right)
\end{aligned}
$$

where:

$$
\begin{aligned}
& M=(H-h) \\
& N=(H-4.5)
\end{aligned}
$$

The Bruce et al. Red Alder Model
3. Bruce et al. Cubic Volume Function:

$$
\begin{aligned}
V & =-K D^{2}(H-4.5)\left[\frac{2 A}{5}\left(P^{5 / 2}-Q^{5 / 2}\right)+\frac{B}{4}\left(P^{4}-Q^{4}\right)\right. \\
& \left.+\frac{C}{33}\left(P^{33}-Q^{33}\right)+\frac{F}{41}\left(P^{41}-Q^{41}\right)\right]
\end{aligned}
$$

where:

$$
\begin{aligned}
P & =[(H-U) /(H-4.5)] \\
Q & =[(H-L) /(H-4.5)] \\
A & =\left[b_{1}\left(10^{-1}\right)+b_{2} D\left(10^{-2}\right)+b_{3} H\left(10^{-3}\right)\right. \\
& +b_{4} D H\left(10^{-5}\right)+b_{5} H^{1 / 2}\left(10^{-3}\right)+b_{6} H^{2}\left(10^{-6}\right) \\
B & =\left[b_{2} D\left(10^{-2}\right)+b_{3} H\left(10^{-3}\right)\right] \\
C & =\left[b_{4} D H\left(10^{-5}\right)+b_{5} H^{1 / 2}\left(10^{-3}\right)\right] \\
F & =\left[b_{6} H^{2}\left(10^{-6}\right)\right]
\end{aligned}
$$

4. Considerations. - Bruce et. al., were the first researchers to develop a profile equation to fit the butt swell at the base of the tree. They accomplished this with a general polynomial model using both fractional and high powers. They conditioned the model to predict a diameter equal to dbh at a height of 4.5 feet and one of zero at the tip of the tree (when h=H). Also, they were the first to use $d^{2}$ as the predictor variable instead of $d$, reasoning that volume estimation is the main purpose of a profile equation and that cubic volume is proportional to the average cross-sectional area of the stem, which $d^{2}$ represents.

The Bruce et. al., model is known to be an accurate model for a wide range of species and tree sizes. The fractional powers and high powers they used were chosen to fit their basic model to the red alder data. For other sets of data, other powers should be investigated. To fit the Bruce et. al., model to other species data, test a range of both fractional powers and high powers, then select those powers that best fit the data.

The Schlaegel Form-Class Model.

1. Profile Model. This form class model is developed from two separate models. The models are conditioned so they join and are equal at a bole height of 17.3 feet, the Girard form class measurement point. It is further conditioned to ensure that predicted diameter (d) equals dbh when the bole height (h) equals 4.5 feet; that $d=D u$, the form diameter, when $h=17.3$ feet; and that $d=0$ when $h=H$.

After conditioning, the complete taper model is:

$$
\begin{aligned}
Y b & =1.0-\frac{\left(D^{2}-D u^{2}\right)\left(X d^{P}-X h^{P}\right)}{D^{2}\left(X d^{P}-X u^{P}\right)} & & \text { for } X u \leq X h \leq 1.0,0 \leq h \leq 17.3 \\
Y t & =(X h / X u)+b_{2} X h(X h-X u) & & \text { for } 0 \leq X h \leq X u, 17.3 \leq h \leq H \\
& +b_{3} X h\left(X h^{2}-X u^{2}\right)+b_{4} X h\left(X h^{3}-X u^{3}\right) & & \\
& +b_{5} X h\left(X h^{4}-X u^{4}\right) & &
\end{aligned}
$$

where:

$$
\begin{aligned}
Y b= & d^{2} / D^{2}, \text { the lower bole model } \\
Y t= & d^{2} / D u^{2}, \text { the upper bole model } \\
H u= & \text { the height of the form measurement point } \\
& (H u=17.3 \text { feet) } \\
X d= & (H-4.5) / H \\
X u= & (H-17.3) / H \\
X h= & (H-h) / H \\
D u= & \text { bole diameter (outside or inside bark) } \\
& \text { measured at } H u
\end{aligned}
$$

2. Solving for $h$ given $d, D, D u$, and $H$. The solution of the profile function for $h$ given $d$ depends on the magnitude of $d$. If $d$ is greater than or equal to Du, then $h$ must be less than or equal to 17.3 feet.

Thus:

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$$
h=H-H\left[X d^{P}-\frac{\left(D^{2}-d^{2}\right)\left(X d^{P}-X u^{P}\right)}{\left(D^{2}-D u^{2}\right)}\right]^{1 / P} \quad \text { for } d>D u
$$

The Schlaegel Form-Class Model.
If $d$ is less than Du, then $h$ must be between 17.3 feet and the tip of the tree, H. Because the top equation does not have a unique solution, use an iterative solution such as the Newton-Raphson technique. The first derivative of the top model Yt with respect to Xh is:

$$
\begin{aligned}
f^{\prime}[Y t(X h)] & =X u^{-1}+b_{2}[2 X h-X u]+b_{3}\left[3 X h^{2}-X u^{2}\right] \\
& +b_{4}\left[4 X h^{3}-X u^{3}\right]+b_{5}\left[5 X h^{4}-X u^{4}\right]
\end{aligned}
$$

3. Schlaegel Cubic Volume Function.
a. Solving for $V$ when both the Lower and upper merchantability limits are greater than or equal to 17.3 feet:

$$
V=K D^{2} H\{(X b-X t)
$$

$$
\left.-\left[\frac{\left(D^{2}-D u^{2}\right)}{D^{2}\left(X d^{P}-X u^{P}\right)}\right]\left[X d^{P}(X b-X t)-\left(\frac{X b^{P+1}-X t^{P+1}}{P+1}\right)\right]\right\}
$$

where:

$$
\begin{aligned}
& X b=(H-L) / H \\
& X t=(H-U) / H \\
& \text { b. Solving for } V \text { when both the lower and upper } \\
& \text { merchantability limits are less than } 17.3 \text { feet: }
\end{aligned}
$$

$$
\begin{aligned}
V & =K D u^{2} H\left\{\left[\left(X b^{2}-X t^{2}\right) / 2 X u\right]\right. \\
& +b_{2}\left[2\left(X b^{3}-X t^{3}\right)-3 X u\left(X b^{2}-X t^{2}\right)\right] / 6 \\
& +b_{3}\left[\left(X b^{4}-X t^{4}\right)-2 X u^{2}\left(X b^{2}-X t^{2}\right)\right] / 4 \\
& +b_{4}\left[2\left(X b^{5}-X t^{5}\right)-5 X u^{3}\left(X b^{2}-X t^{2}\right)\right] / 10 \\
& \left.+b_{5}\left[\left(X b^{6}-X t^{6}\right)-3 X u^{4}\left(X b^{2}-X t^{2}\right)\right] / 6\right\}
\end{aligned}
$$

### 33.32 - Exhibit 10 -- continued

C. Solving for $V$ when $L<17.3$ feet and $U>17.3$ feet. In this case find both the volume in the bottom of the tree (Vb) from L up to 17.3 feet, and in the top of the tree (Vt) from 17.3 feet up to U , then add Vb and Vt to estimate bole volume. First, find the volume in the bottom of the tree (Vb) by letting $\mathrm{Xb}=(\mathrm{H}-\mathrm{L}) / \mathrm{H}$ and Xt $=(\mathrm{H}-17.3) / \mathrm{H}$ and solve for the volume to 17.3 feet using the equation as in $3 a$ above. Next, find the volume in the top of the tree above 17.3 feet (Vt) by using equation as in 3 b above, letting $\mathrm{Xb}=(\mathrm{H}-17.3) / \mathrm{H}$ and Xt $=(H-U) / H$. Then bole volume between $L$ and $U$ is: $V=$ $\mathrm{Vb}+\mathrm{Vt}$.

## 4. Considerations.

a. The form class profile model was developed for use in hardwoods and was developed and tested for willow oak and proved to be superior to the Max-Burkhart model in all categories of testing, whether by diameter classes, total height classes, relative height classes, or combinations of the three.
b. Even though it was developed for hardwoods, it is expected to perform well for conifers.
c. Use of this model requires an extra measurement, Du, at 17.3 feet. Estimating form diameter is an extra expense in timber cruising, but it provides a more accurate tree volume estimate. However, it reduces the variation between trees and the number of trees to be measured, which largely compensates for the cost of the extra diameter measurement.
d. In some situations, it may be sufficient to use a fixed form class for a species in a given location, or to set a form class for that species/location as a function of dbh, height, or both. In these cases, substitute local constants for Du, which negates the need to measure Du in the field.
e. The form class model allows an opportunity to greatly expand the usable range of an individual profile equation.
f. Trees having the same $d b h$ and height have different volumes due to differences in bole shape (also known as taper or form). As form class changes by a single point (say 80 to 81 ), the volume changes by approximately 3 percent. Thus, much of the variation that exists in current volume and profile functions is due to assuming an average tree form.
g. Standard volume and profile equations apply to a limited geographic or physiographic area because of the changes in tree form that exist over large areas and therefore, a number of volume and profile equations are developed for a single species to improve volume estimates in a localized area. A form class profile equation should be valid over a larger area.

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h. A plot of a two-equation form class model is shown in exhibit 11.

```
            TIMBER VOLUME ESTIMATOR HANDBOOK
                                    33.32 - Exhibit 11
Plot of a two-equation form taper model with constant dbh
        and total height with different form diameters.
```



1. Profile Model:

$$
\begin{aligned}
\frac{d^{2}}{D^{2}} & =b_{1}(X-1)+b_{2}\left(X^{2}-1\right) \\
& +b_{3}\left(a_{1}-X\right)^{2} I_{1}+b_{4}\left(a_{2}-X\right)^{2} I_{2}
\end{aligned}
$$

where:

$$
\begin{aligned}
X= & h / H \\
a_{i}= & \text { join points estimated from the } \\
& \text { data; } a_{2} \text { is the lower point, } \\
& \text { and } a_{1} \text { is the upper } \\
I_{1}= & 1 \text { when } X \leq a_{1} \\
= & 0 \text { when } X>a_{1} \\
I_{2}= & 1 \text { when } X \leq a_{2} \\
= & 0 \text { when } X>a_{2}
\end{aligned}
$$

Thus, the taper equation in the bole below the bottom join point ( $\mathrm{X} \leq \mathrm{a}_{2}$ ) is:

$$
\begin{aligned}
\frac{d^{2}}{D^{2}} & =b_{1}(X-1)+b_{2}\left(X^{2}-1\right) \\
& +b_{3}\left(a_{1}-X\right)^{2}+b_{4}\left(a_{2}-X\right)^{2}
\end{aligned}
$$

In the middle portion of the bole above $a_{2}$ and below $a_{1}$, i.e $a_{2}<X \leq a_{1}$, the profile equation is :

$$
\frac{d^{2}}{D^{2}}=b_{1}(X-1)+b_{2}\left(X^{2}-1\right)+b_{3}\left(a_{1}-X\right)^{2}
$$

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The top portion of the bole above $a_{1}\left(a_{1}<X\right)$ is:

$$
\frac{d^{2}}{D^{2}}=b_{1}(X-1)+b_{2}\left(X^{2}-1\right)
$$

The Max-Burkhart Model.
2. Solving for $h$ given $d$. Since the Max-Burkhart model consists of three joined 2nd degree polynomials, solve for h given $d$ by solving the quadratic equation for one of the profile equation segments, depending on where $d$ is located in relation to the diameters of the join points:

$$
h=H\left[-B-\left(B^{2}-4 A C\right)^{1 / 2}\right] / 2 A
$$

where:

$$
\begin{aligned}
A= & b_{2}+P_{1} b_{3}+P_{2} b_{4} \\
B= & b_{1}-2 P_{1} a_{1} b_{3}-2 P_{2} a_{2} b_{4} \\
C= & -b_{1}-b_{2}-\frac{d^{2}}{D^{2}}+P_{1} a_{1}^{2} b_{3}+P_{2} a_{2}^{2} b_{4} \\
P_{1}= & 1 \text { if } d \geq d_{1} \\
= & 0 \text { if } d<d_{1} \\
P_{2}= & 1 \text { if } d \geq d_{2} \\
= & 0 \text { if } d<d_{2} \\
d_{1}= & D\left[b_{1}\left(a_{1}-1\right)+b_{2}\left(a_{1}^{2}-1\right)\right]^{1 / 2} \\
= & \text { estimated bole diameter at upper } \\
& j o i n \text { point, }\left(h=a_{1} H\right) \\
d_{2}= & D\left[b_{1}\left(a_{2}-1\right)+b_{2}\left(a_{2}^{2}-1\right)+b_{3}\left(a_{1}-a_{2}\right)^{2}\right]^{1 / 2} \\
= & \text { estimated bole diameter at lower } \\
& \text { join point, (h=a } \left.a_{2}\right)
\end{aligned}
$$

The Max-Burkhart Model.
3. The Max-Burkhart Cubic Volume Function. The Max-Burkhart equation is as follows:

$$
\begin{aligned}
V & =K D^{2} H\left\{\frac{b_{2}}{3}\left(X^{3}-Y^{3}\right)+\frac{b_{1}}{2}\left(X^{2}-Y^{2}\right)-\left(b_{1}+b_{2}\right)(X-Y)\right. \\
& -\frac{b_{3}}{3}\left[\left(a_{1}-X\right)^{3} I_{1}-\left(a_{1}-Y\right)^{3} J_{1}\right] \\
& \left.-\frac{b_{4}}{3}\left[\left(a_{2}-X\right)^{3} I_{2}-\left(a_{2}-Y\right)^{3} J_{2}\right]\right\}
\end{aligned}
$$

where:

$$
\begin{aligned}
X & =U / H \\
Y & =L / H \\
I_{1} & =1 \text { if } X \leq a_{1} \\
& =0 \text { if } X>a_{1} \\
I_{2} & =1 \text { if } X \leq a_{2} \\
& =0 \text { if } X>a_{2} \\
J_{1} & =1 \text { if } Y \leq a_{1} \\
& =0 \text { if } Y>a_{1} \\
J_{2} & =1 \text { if } Y \leq a_{2} \\
& =0 \text { if } Y>a_{2}
\end{aligned}
$$

The Max-Burkhart Model.
4. Considerations.

```
a. Consider the Max-Burkhart model to be a very
flexible, two-variable, general purpose model and
typically, the model used as a basis for comparing newly
developed models.
b. Consider this model where accuracy is deemed
important. The Max-Burkhart profile model has been
accepted as one of the most accurate and flexible
two-variable models available.
c. The model is relatively complex, hard to understand,
difficult to fit to the data, and hard to use.
d. The model consists of three parabolic equations
connected at two locations called "join points".
Condition the equations so that as the taper line is
smooth and continuous from one parabolic equation to the
next. The taper line is considered to be smooth at the
join point if the slopes of the two equations are equal
and continuous if equal at the join point. Also,
condition the three parabolic equations to force the
taper line through dbh and the tree tip.
e. Consult references on the method such as Jeff Martin
    (sec. 08).
```

```
34 - DIRECT VOLUME ESTIMATION--VOLUME EQUATIONS. Prepare direct
estimators that predict tree volume from dbh and height, from dbh
alone, or from dbh, height, and some expression of tree form. If
tarif tables are preferred, predict volumes from dbh and tarif number
which is an index which incorporates the additional effects of both
height and form. Use tree height as a predictor. Total height is
appropriate for most conifers, or identify an appropriate
merchantable height which is the most useful expression of height for
some hardwoods. As with stem profile equations, estimate
coefficients of volume equations by regression techniques from a
sample of trees for which volume has been accurately measured.
Design direct estimators to predict total stem volume or merchantable
volume between the stump and the specified merchantable top limit.
34.1 - Tree Volume Models. Consider total volume to be the cubic
content of the tree. Recognize that board feet, cords, and other
product measures are not volume. Ensure that when such terms are
applied to unprocessed trees or logs they are a statement of product
potential, not stated as a volume estimate.
A tree volume model relates measured dimensions of a tree stem to the
volume contained in the stem. Recognize that a mathematical model is
only an approximation to the actual geometric solid which is a tree
stem. However, quite simple models are adequate to predict volume in
many circumstances.
Apply the notation given in exhibit 01 for the direct volume
estimation formulae in this section.
```

```
            TIMBER VOLUME ESTIMATOR HANDBOOK
                    34.1 - Exhibit 01
Notation for Direct Volume Estimation - Volume Equations
V = total volume of a tree stem, from ground level to
                tip.
V
V
V
        merchantability limit.
D = tree diameter at breast height, outside bark.
D = stump diameter at cut surface.
D
Du = an upper stem diameter, inside or outside bark as
        stated.
H = total tree height.
H
L
R
k = п/(4*144) = 0.005454154
bi}== estimated coefficient
```

34.12 - Combined Variable Model. Use the Combined Variable Model only if the cylinder form factor is nearly the same for all trees regardless of their size. Recognize that this may not be true even if trees of all sizes had exactly the same shape, because breast height is not the same relative point on trees of different sizes. Consider this model when the range of tree sizes in the population is not great. However, consider that estimators based upon this model will, in theory, be biased in differing direction and magnitude at different points in the domain of $D^{2} H$.

The most common model used for total stem volume is the Combined Variable Model (Section 08, Spurr, 1952):

$$
\mathrm{V}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}^{2} \mathrm{H}
$$

Ensure that when estimated from data, $b_{0}$ is very small
(theoretically, however, it should not equal zero) and $b_{1}$ is nearly proportional to the data set's mean ratio of actual tree volume to volume of a cylinder with the same diameter and height. This ratio is called the cylinder form factor (CFF). The constant of proportionality is $k$. Estimate the coefficients $b_{0}$ and $b_{1}$ using ordinary least squares (OLS) regression techniques.

Variance about the regression is not homogeneous and is correlated with tree size. Therefore, use observation weights proportional to $\left(D^{2} H\right)^{-n}$ when fitting this equation form. If such weighting is not done, estimates of parameters will be unbiased, but they will not be the best linear unbiased estimators (BLUE) and will not be minimum variance unbiased linear estimators.

The value of $n$ which best describes residual variance may be as low as 0.5 or as large as 2. Use any value of $n$ on the interval [1.5,2] which can be expected to give reasonably good results (better than $\mathrm{n}=0$ which implies equal observation weights). Do not use values of n outside [1.5,2] which are unlikely to be optimal.

Employ weighting functions depending upon the regression software used and user preference. For example, some regression software allows defining the value of a variable which is to be used as the observation weight. Some programs go further and scale the weights so that the sum of weights is equal to the number of observations in the sample. Others require the constant of proportionality to be adjusted by the user in order to meet this condition. If software being used does not have the facility for defining a weight variable, multiply both sides of the model by the square root of the weighting function. That is, multiply through by $\left(D^{2} H\right){ }^{-n / 2}$. This results in a transformed model, but one with the same coefficients as the original model. Use the estimated coefficients, which should be nearly BLUE, in the original untransformed model.

Do not ordinarily construct a volume estimator based on the Combined Variable model alone. In most cases, construct one with one set of coefficients for trees no larger than an arbitrary size, and a different set for trees larger than that size to accommodate form factor differences between trees of different sizes. Such models have an abrupt change in the relationship of volume to dbh and height at the arbitrarily chosen size and are no longer widely recommended. When reasons of expedience justify the practice, define the point at which the two regression lines intersect (the "join point") in terms of the variable $D^{2} H$, not in terms of $D$ alone. Select the join point

## TIMBER VOLUME ESTIMATOR HANDBOOK

that, jointly with other coefficients, minimizes the sum of squared residuals. Do not select it separately and arbitrarily.
34.13 - Adjustments to the Combined Variable Model. Because the Combined Variable Model's constant form factor assumption is not applicable over a reasonable range of tree sizes, it may be desirable to add linear terms to the model which allow cylinder form factor to vary with tree size. Consider one of the many possible examples such as the Australian Model (Stoat, 1945):

$$
\mathrm{V}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}^{2} \mathrm{H}+\mathrm{b}_{2} \mathrm{DH}
$$

The cylinder form factor can vary with tree size. Do this by dividing the equation by $\mathrm{kD}^{2} \mathrm{H}$, thus:

$$
C F F=V\left(k D^{2} H\right)^{-1}=b_{0}\left(k D^{2} H\right)^{-1}+b_{1} k^{-1}+b_{2}(k D)^{-1}
$$

Since $b_{0}$ is very small, the first term right of the second equality has negligible effect on CFF. If $b_{1}>0$ and $b_{2}>0$, what remains of the expression describes a CFF which decreases with tree dbh (more rapidly at first then more slowly at larger dbh) asymptotic to a limiting CFF, $b_{1}$. This is consistent with the relation of form factor to tree size. However, the rate of change in CFF with change in dbh may not be well described by the reciprocal term for all of the dbh range. No integer value for the negative exponent of $D$ may adequately describe the relationship between CFF and D over the entire range of $D$. In fact, no single exponent (whether integer or not) may be sufficient. Also, form factor is related to tree height as well as to dbh--perhaps to the ratio H/D. If poor results are obtained consider form factor models.
34.14 - Form Factor Models. In these models, calculate cylinder form factor (CFF) $C F F=V\left(k D^{2} H\right)^{-1}$ for each tree in the volume estimator sample. Use CFF as the dependent variable in deriving an appropriate form factor model and estimating coefficients. Estimate coefficient values by ordinary least squares (OLS) regression. Consider CFF as a dimensionless number which reduces concern for strict maintenance of cubic dimensionality in the model. Also, a model with volume as the dependent variable exhibits heteroskedasticity. Since CFF is nearly homoskedastic, do not use observation weights in estimating coefficients. An example of a model describing the relation of CFF to D and H is:

$$
\mathrm{CFF}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}^{-1}+\mathrm{b}_{2} \mathrm{D}^{-2}+\mathrm{b}_{3} \mathrm{H}+\mathrm{b}_{4} \mathrm{H}^{2}+\mathrm{b}_{5}(\mathrm{H} / \mathrm{D})+\mathrm{b}_{6}(\mathrm{H} / \mathrm{D})^{2}
$$

Volume is then estimated for any tree by:

## $\mathrm{V}=\mathrm{kCFF} \mathrm{D}^{2} \mathrm{H}$

The CFF model in this example contains polynomial and polynomial-reciprocal terms, but some CFF equations have contained terms as high as the fifth degree. Terms with fractional exponents may be included, as well as those with integer exponents. Avoid any set of independent variables that exhibits strong multicollinearity which may generate a moment matrix too close to singular for sufficiently accurate inversion.

Use a stepwise regression procedure to avoid problems with a singular matrix since variable sets with excessive multicollinearity are not selected. Assess the terms that might be included in the model that

## TIMBER VOLUME ESTIMATOR HANDBOOK

are likely to have predictive value using analysis of variance from regression. Use a stepwise regression procedure that is included in most statistical packages. Set $F$-to-enter and $F$-to-delete values at 3.99 and 4.00 , respectively. The programmed stepwise algorithm results in a fitted regression equation which contains those, and only those, variables significant at or beyond the 0.05 probability level.

When volume is the dependent variable, most models will appear to have a precise fit with coefficients of determination ( $\mathrm{R}^{2}$ ) in the range 0.95-0.98. However, when CFF is used as the dependent variable do not expect such precise fits. Expect values of $\mathrm{R}^{2}$ in the range 0.20-0.40. The greater the range of tree sizes in the sample, the greater the variation in CFF, and thus more of it is explained by regression. Consider a coefficient of determination in the neighborhood of 0.40 to be a good fit. Do not compare precision of two fitted regressions, one using CFF as the dependent variable, and the other volume as the dependent variable since two different things are being compared. Compare the two, using volume residuals from the CFF estimator computed as:

```
residual = (V - k CFF D'H)
where V = observed volume of an individual sample tree.
CFF = cylinder form factor predicted for the
                same tree by the fitted estimating equation.
```

Square and sum the residuals computed in this way over the sample to obtain a sum of squared residuals or other derived statistics. These are comparable to the sum of squared residuals obtained from fitting an equation with volume as the dependent variable.

Since summary measures of goodness-of-fit such as $R^{2}$ or the root mean square residual (RMS) seldom convey enough information by themselves to allow a thorough comparison among fitted equations or of a particular fit to an objective norm, use analysis of residuals described in section 35 .

Polynomial-reciprocal models fit data quite well within the range of the sample, but often predict wildly outside the domain of
independent variables represented by the sample data. Therefore, if the sample covers all tree sizes to be encountered in practice, expect no problem. Because this is not likely, extend the trend of the estimator and the CFF estimator beyond the sample data cautiously and only when it is a practical necessity. Use a linear equation rather than the polynomial-reciprocal CFF estimator outside the domain of independent variables represented by the sample. Find coefficients of the linear equation by locating two suitable points graphically and effecting a simultaneous solution.

The polynomial-reciprocal model has a high degree of flexibility and may allow unrealistic and undesirable changes of slope on the CFF surface even within the range of the sample. Check for an abnormally formed tree included in the sample, a mistake in the observed values of the dependent or independent variables, or anything leading to an exceptionally large difference between observed and regression-predicted values. For this reason conduct an analysis of residuals to detect local irregularities in the regression surface.
34.15 - Allometric or Log-Linear Models. This model is similar to the Combined Variable Model:

$$
\mathrm{V}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}^{2} \mathrm{H}
$$

If $b_{0}$ is ignored because it is so small as to be truly negligible (for example, volume of a tree 4.5 feet tall), $b_{1}$ is left as the sole expression of tree form. Do not accept the idea of a constant cylinder form factor (CFF). If it appears the polynomial-reciprocal estimator for CFF may give problems, consider a different representation of CFF (or $\mathrm{b}_{1}$ ). The ratio of a tree's total height to its dbh must be related to its CFF. The height to diameter ratio (H/D) is the inverse of average taper between breast height and the tree's tip. If CFF were a simple multiple of $H / D$, such as CFF = $q^{\prime}(H / D)$, then a volume model could be written

$$
V=k q(H / D) D^{2} H=k q D H^{2}
$$

This model is dimensionally correct. Tree volume is the cross-sectional area ( $\mathrm{kD}^{2}$ ) accumulated up the stem. The accumulation of $\mathrm{kD}^{2}$ is a linear function of H . Consider a more flexible relationship between CFF and H/D, for example:

$$
C F F=H^{b} / D^{c}
$$

allows writing

$$
\begin{aligned}
V & =k \operatorname{CFF} D^{2} H \\
& =k\left(H^{b} / D^{c}\right) D^{2} H
\end{aligned}
$$

which by combining exponents becomes

$$
\mathrm{V}=\mathrm{k} \quad \mathrm{D}^{2-\mathrm{c}} \mathrm{H}^{1+\mathrm{b}}
$$

or, letting $\mathrm{b}_{1}=2-\mathrm{c}$ and $\mathrm{b}_{2}=1+\mathrm{b}$,

$$
V=k D^{b_{1}} H^{b_{2}}
$$

Use one of two ways to fit this to sample data. Choose which to use based on the distribution of error associated with the model, not upon the fact that it is easier to fit a linear regression. If random
error e is additive, write the model with error included as:

$$
V=k D^{b_{1}} H^{b_{2}}+e
$$

then estimate coefficients $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$ by nonlinear regression. Do not make transformations by taking logarithms to render coefficients amenable to estimation by linear regression. Previous discussion in this section is based on assumption that error is additive. Do not assume the Combined Variable Model has additive error, and that the allometric model does not simply because the exponents of $D$ and $H$
differ slightly from 2 and 1 , respectively. If the model with additive error is fitted in the form given, use observation weights proportional to $\left(D^{2} H\right)^{-n}$. Depending upon the nonlinear regression software used, weighting may need to be done by dividing the equation by $D^{2} H$, thus arriving at an allometric form factor model. Fit this allometric model to data, whether with form factor or volume as the dependent variable, and expect to find that $b_{1}$ is not exactly equal to 2 , nor is $b_{2}$ exactly equal to 1 . Do not expect equality if the concept of form factor changing with tree size is accepted. It may also be that $\mathrm{b}_{1}+\mathrm{b}_{2}$ is not exactly equal to 3; that is, that strict cubic dimensionality is not maintained. Expect the sum of exponents to be near 3 and the small difference may not matter. However, if strictly logical dimensionality is desired, maintain it by writing the model:

$$
V=k b_{1} D^{b_{2}} H^{\left(3-b_{2}\right)}+e
$$

Recognize that the reduction in precision of fit suffered by insistence upon strict cubic dimensionality will be small, and a significant benefit may be much quicker convergence of the fitting algorithm. The sum-of-squares surface generated by an allometric volume model with independent exponents possesses a long trough in the space of the exponents. Expect repeated iterations of the algorithm to move along the bottom of this trough with small reduction in the sum of squares, but enough not to terminate the fitting process. Expect optimality to be reached very slowly. Reduce the parameter space by maintaining a strictly cubic model changes the sum-of-squares surface to one that exhibits a well defined minimum point rather than a long trough with little gradient.

If multiplicative error is suspected, then write the model including the error term as:

$$
V=k b_{1} D^{b_{2}} H^{b_{3}} e
$$

then, make the model linear by taking logarithms resulting in:

$$
\ln V=\ln \left(k b_{1}\right)+b_{2} \operatorname{lnD}+\mathrm{b}_{3} \ln H+\ln (e)
$$

If strict cubic dimensionality is to be maintained use the model:

$$
\left(\operatorname{lnV}-\mathrm{b}_{3} \ln H\right)=\ln \left(\mathrm{k} \mathrm{~b}_{1}\right)+\mathrm{b}_{2}(\operatorname{lnD}-\ln H)+\ln (\mathrm{e})
$$

Fit linear models by OLS techniques. This justifies logarithmic transformation in the majority of cases where it is used. No consideration is given to the distribution of error, and logarithmic transformation is assumed to have a self-weighting effect. This means that heteroskedasticity (probably strongly exhibited by the untransformed model) is reduced in the log-linear model. It will be even more reduced where the dependent variable is (lnV - $\mathrm{b}_{3} \operatorname{lnH}$ ), but will not be eliminated. Recognize that this is not the only effect of transformation on the error distribution.

Recognize that it is not consistent to claim additive error for models with $V$ as the dependent variable and then claim additive error for models which use $\operatorname{lnV}$ as the dependent variable. However, the difference in estimates resulting from difference in error distribution assumptions may not be very great in practical applications.

Do not overlook the bias attached to estimates of $V$ of logarithmic transformation and make needed correction for bias. For additional reference see Bradu and Mundlak (1970). The necessary bias correction factor (BCF) is approximately:

$$
B C F=\exp \left(S_{y x}^{2} / 2\right)
$$

where
$S_{y x}^{2}=$ the variance of $\operatorname{lnV}$ residuals about the regression surface.

Although correction should be made in estimating tree volumes, it may not be essential. The absence of large errors resulting from this omission is due to the characteristically small root mean square residual of $\operatorname{lnV}$. Expect the correction factor to change estimates by no more than five percent of predicted volume. Recognize that such small bias is usually obscured by errors of diverse provenances in the volume estimation process.
34.16 - Form Class or Form Quotient Estimators. Consider making volume estimators more precise by including a variable to take into account the different form of each individual tree, or of an identifiable group of trees which differ in form from the average. To do this, use a stem diameter as the additional variable $\mathrm{D}_{\mathrm{u}}$, measured at some point above breast height. Incorporate $D_{u}$ in the model through the form quotient, $D_{u} / D$.

For the form quotient to have any meaning from one tree to another, measure $D_{u}$ at the same height above ground, or the same relative height, on all trees. Evaluate defining this height differently for different kinds of form quotients to see if one is more useful than another. Measuring $D_{u}$ at half the tree's total height yields Jonson's form quotient. Another approach is measuring $D_{u}$ at one-fifth the tree's total height. The most familiar expression of tree form is Girard form class, which is $100 * D_{u} / D$, with $D_{u}$ measured inside bark at the top of the butt log, 17.3 feet above ground line. Use a device called the Wiant Wedge (FSH 2409.12) to make measurement of Girard form class possible without measuring $D_{u}$.

Use form class or form quotient as an independent variable with a form factor model. Consider form quotient one of the most important variables.

Weigh the value of precision gained by use of form class or form quotient estimators against the additional cost of using them. Additional cost comes from the need to obtain a measure of form for each tree to which the estimator is applied. This may mean measurement of an upper stem diameter, which is more expensive than measuring dbh. The use of an instrument such as the Wiant Wedge does not ensure that the estimate of form class obtained is the same as that obtained by direct measurement of $D_{u}$. Obtain form class or quotient for felled estimator sample trees by direct measurement of $D_{u}$. Evaluate form class for better or poorer results than a standard

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volume estimator based upon $D$ and $H$ alone by evaluating the following:

1. The difference between mean form class of trees in the cruise area and the mean in the sample from which the standard volume estimator was constructed. If they are very nearly the same, the standard volume estimator will be as good as the use of a form class estimator with even a good mean form class estimate.
2. The change in average form class from one tree size to another is implicit in the standard estimator. If a single average form class based on a sample of climbed, felled, or windthrown trees is used for trees of all sizes, expect the volume estimates for larger and smaller trees to be somewhat biased. If the form class sample covers a sufficient range of tree sizes, it may be possible to characterize the relation between tree size and form class to avoid such bias. However, this complicates use of a form class estimator and adds to its cost.
3. The accuracy with which the sample mean form class represents the mean form class in the cruise area. If this is poor, results of using a form class estimator will be poor.
34.17 - Tarif Volume Estimators. Tarif tables are a comprehensive set of local volume tables indexed by tarif number. Do not construct new tarif tables. To use existing tables, establish the appropriate tarif number for a particular stand and select the corresponding tarif table. Measure only the diameter breast height (dbh) of cruised trees to enter the local volume table. Carefully determine the tarif number. Fell and measure a sample of trees to determine which tarif table best predicts their volumes. If the stand is small, consider measuring all tree heights and using a standard volume estimator. Use access tables based on the height dbh relationship if they have been prepared. Using these access tables, take only a sample of height measurements to estimate tarif number.

Do not use a different tarif number for each dbh class in the same evenaged stand, since it is contradictory to the theory of tarif table construction and use.
34.2 - Merchantable Volume. Most uses for volume estimates require estimates of merchantable volume rather than total volume. Three approaches to modeling merchantable volume are described in the following sections.
34.21 - Simple Approach. To use this approach, use the same model as for total volume, substituting $\mathrm{V}_{\mathrm{m}}$ as the dependent variable. Recognize that the approach takes no account of the logical relationships involved and may be inaccurate. Use this method only when other approaches do not meet the needs.
34.22 - Merchantable Volume Ratio Model. This method may be used to ensure that estimates of $\mathrm{V}_{\mathrm{m}}$ are less than V . The merchantable volume estimator is then:

$$
\mathrm{V}_{\mathrm{m}}=\mathrm{R}_{\mathrm{m}} \mathrm{~V}
$$

where $R_{m}$ is zero for trees of less than merchantable size, becomes immediately greater than zero as trees reach merchantable size, and from that point increases monotonically with tree size asymptotic to unity. As a practical matter, over the range of possible tree sizes,
$R_{m}$ will be somewhat below its theoretical asymptote. For each sample tree an observed value of $R_{m}$ is $V_{m} / V$. Possible models relating $R_{m}$ to tree dimensions are:

$$
\begin{array}{ll}
\text { 1. } & R_{m}=1-q \exp (-X B) \\
\text { 2. } & R_{m}=1-b / D \\
\text { 3. } & R_{m}=1-b / D^{2} \\
\text { 4. } & R_{m}=1-b /\left(D^{2} H\right)
\end{array}
$$

where $b$ and $q$ are model coefficients, $\mathbf{X B}$ is a generalized function of D and/or $H$, linear with respect to $\mathbf{B}$, that increases with tree size. Use other possible models that exist, but ensure the model used has the property of increasing monotonically with tree size and never exceeding unity. Example models 2 through 4 are ratio estimators. Estimate the coefficients, b, by regressions fitted through the origin with (1 - $R_{m}$ ) as the dependent variable. The observation weights depend upon the nature of variance in (1- $R_{m}$ ) about regression.

In example model 1 above, estimate $q$ and $B$ by OLS with the log-linear model

$$
\ln \left(1-R_{m}\right)=\ln (q)-X B
$$

Find the best specification for $\mathbf{X B}$ by experimentation with various transformations and interactions of $D$ and $H$. The only requirement is that it increase in magnitude with increasing tree size. Using a log-linear model may seem at odds with advice about error distribution given in section 34.15. If error associated with all of the previously mentioned models is additive, then the logarithmic transformation is not appropriate. However, to partially address the problem, fit the log-linear equation as one step to arrive at a suitable transformation of the information about tree dimensions. Fit the equation:

$$
\mathrm{R}_{\mathrm{m}}=\mathrm{b}_{0}+\mathrm{b}_{1} \exp (-\mathbf{x B})
$$

by OLS techniques consistent with the assumption of additive error implicit in the other models suggested above. If $\mathrm{b}_{0}<0.95$ or $\mathrm{b}_{0}>1$ then $b_{0}$ should be restricted by fitting:

$$
\left(1-R_{m}\right)=-b_{1} \exp (-X B)
$$

Coefficients in the vector $\mathbf{B}$ do not have desirable statistical properties. However, the $\exp (-\mathbf{X B})$ is a fair transformation of $D$ and H measurement, and is probably better than the reciprocals or squared reciprocals.
34.23 - Merchantable Volume Difference Model. Consider this model if only merchantable volume needs to be estimated. Calculate merchantable volume as total volume less those portions of the tree which are unmerchantable. Use the algebraic relationship:

$$
V_{\mathrm{m}}=\mathrm{V}-\mathrm{V}_{\mathrm{s}}-\mathrm{V}_{\mathrm{t}}
$$

If a satisfactory estimator for total volume, $V$, already exists, model $\mathrm{V}_{\mathrm{s}}$ and $\mathrm{V}_{\mathrm{t}}$ and estimate the model parameters. $\mathrm{V}_{\mathrm{s}}$ is a function
of $\mathrm{D}_{\mathrm{s}}$, stump height, and stump shape. Expect stump shape to be different for different sized trees. Fix stump height at some height related to utilization practices, such as one foot. Determine if a linear relationship between $D^{2}$ and stump volume exists. Look for the relationship between $D^{2}$ and $D_{s}{ }^{2}$, stump height, and stump shape to be contained in the coefficient of $D^{2}$. Expect some linear effect of $D$ in the stump volume model, so use, as a beginning:

$$
\mathrm{V}_{\mathrm{s}}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}+\mathrm{b}_{2} \mathrm{D}^{2}
$$

If the linear term is not useful, delete it.
Assume the top portion of most merchantable-sized trees closely resembles a cone. Then, volume of the top is nearly:

$$
V_{t}=(1 / 3) \mathrm{k} \quad\left(D_{m}\right)^{2} L_{t}
$$

where $L_{t}$ is length of the unmerchantable top.
In any particular application $D_{m}$ is fixed, not a variable, so express $L_{t}$ in terms of $D$ and $H$. $L_{t}$ is longer for smaller, younger trees than for older, larger ones. Expect that $L_{t}$ is related to average taper of the tree. (H/D) is the inverse of the tree's average taper. If $\mathrm{L}_{\mathrm{t}}$ is linearly related to (H/D) as:

$$
L_{t}=C_{0}+C_{1}(H / D)
$$

then express nonmerchantable top volume as:

$$
\mathrm{V}_{\mathrm{t}}=(1 / 3) \mathrm{k} \mathrm{D}_{\mathrm{m}}^{2}\left[\mathrm{C}_{0}+\mathrm{C}_{1}(\mathrm{H} / \mathrm{D})\right]
$$

or, after condensing constants as:

$$
V_{t}=b_{0} D_{m}^{2}+b_{1} D_{m}^{2}(H / D)
$$

Expect that a simple reciprocal expression of $D$ is not always sufficient. For many southwestern species, consider a negative exponent of 1.5 . Call this exponent $n$ for generality and combine total volume, stump volume, and top volume into one expression:

$$
V_{m}=V-\left(b_{0}+b_{1} D+b_{2} D^{2}\right)-\left(b_{3} D_{m}^{2}+b_{4} D_{m}^{2} H D^{-n}\right)
$$

Estimate coefficients (except $n$ ) by moving $V$, a previously determined value, to the left side of the equation. Select the exponent $n$ by experiment. Drop terms which turn out to have no statistical or practical effect from the estimating equation.
34.24 - Use of Merchantable Height. Determine if the expected users measure merchantable height of a tree. If so, decide how to model merchantable volume depending upon whether or not total height is measured also. If it is not, expect estimation of total volume to be more difficult. If both total and merchantable heights were measured, top volume might become:

$$
V_{t}=(1 / 3) \mathrm{k} D_{m}^{2}\left(H-H_{m}\right)
$$

and the merchantable volume difference model is:

$$
V_{m}=V-\left(b_{0}+b_{1} D+b_{2} D^{2}\right)-b_{3} D_{m}^{2}\left(H-H_{m}\right)
$$

If the merchantable volume ratio approach is used, expect the ratio $\mathrm{V}_{\mathrm{m}} / \mathrm{V}$ to be related to the ratio $\mathrm{H}_{\mathrm{m}} / \mathrm{H}$, though not linearly. Starting with the relationship:

$$
V_{m} / V=b_{1}\left(\frac{H_{m}}{H}\right)^{b_{2}}
$$

and allow individual exponents for $H_{m}$ and $H$ :

$$
V_{m} / V=b_{1} H_{m}^{b_{2}} H^{b_{3}}
$$

Estimate the exponents $\mathrm{b}_{2}$ and $\mathrm{b}_{3}$ by transformation to log-linear form. If error about the model above is multiplicative, estimate $\mathrm{b}_{1}$ from its logarithm in the log-linear form fitted by OLS. Otherwise, after obtaining values of the exponents from fitting the log-linear form $b_{1}$ and $b_{0}$ if it is expected to have a nonzero value, estimate by:

$$
V_{m} / V=b_{0}+b_{1} H_{m}^{b_{2}} H^{b_{3}}
$$

Restrict $b_{0}$ to zero if necessary. In this case, do not regard exponents as maximum likelihood estimators, or as possessing any other desirable statistical properties other than being better exponents for transforming $H_{m}$ and $H$ than an arbitrarily chosen integer would be. However, they are not arbitrary since they were derived from the available data. Count them for one degree of freedom in any assessment of precision that is undertaken.
34.25 - Use of Merchantable Top Diameter as a Predictor Variable. In the merchantable volume ratio model (sec. 34.22) and the merchantable volume difference model (sec. 34.23), assume that when $\mathrm{V}_{\mathrm{m}}$ is volume to a certain top diameter $\mathrm{D}_{\mathrm{m}}$, then $\mathrm{D}_{\mathrm{m}}$ is fixed in a particular fitting of the model to data. Treat $D_{m}$ as a variable in fitting the model, thereby achieving a smooth and transitive relationship between merchantable volumes $\mathrm{V}_{\mathrm{m}}$ associated with a range of top diameters $\mathrm{D}_{\mathrm{m}}$.

Take a sample large enough so that different observations of $D_{m}$ and $V_{m}$ are taken on different trees, and make no more than one observation of $\mathrm{D}_{\mathrm{m}}$ and $\mathrm{V}_{\mathrm{m}}$ on each sample tree. This avoids correlated residuals about the regression surface. Recognize that one of the necessary conditions for best linear unbiased estimators (BLUE) is that residuals be independent.

Expect several merchantable volumes $\mathrm{V}_{\mathrm{m}}$ to be calculated to corresponding top diameters $D_{m}$ on each tree in the sample. This results in highly correlated residuals about regression and violates the BLUE assumptions. This is a situation with time series and cross-section effects in the same sample. Use Generalized Least Squares (GLS), instead of OLS, to derive BLUE from such data. Meeting the BLUE assumptions may not be crucial since parameter estimates will be unbiased, but estimates of variances will be biased. Therefore, hypothesis tests will be invalid. This may not matter if the structure of a model is well known from theory. Tests
of hypotheses, such as for differences between geographic subregions, even though appropriate, may not be needed and are not valid.
34.3 - Comparison of Direct Estimators with Stem Profile Estimators. Avoid the use of direct volume estimators except as a check of stem profile estimators. Prepare and use stem profile estimators unless the Regional Forester approves the justification to use another method. Comparison may involve a set of volumes or estimators, especially when several merchantability standards are involved and consistency of volume estimates between them must be maintained. Make necessary assumptions about the shape of the tree, and expect biases that are difficult to detect.

35 - VERIFICATION OF ESTIMATORS. Verify estimators with the data on which they are based to eliminate (1) lapses in programming logic,
(2) flaws in algorithms, and (3) bias in computations. Verify using one or more of the tests in the following sections. Do not confuse this process with validation which tests how well the estimator predicts using an independent data set.
35.1 - Use of Standard Statistical Measures. Check the standard statistical measures of goodness of fit reported by the computer regression programs used in developing an estimator. Use these in choosing the mathematical models, but consider them less useful in verifying the estimator program. Be aware that these measures often have no single narrow interpretation. Check other valid evidence including performance with other data sets, consistency of significance in subsets of the present data, and biological or mechanical reasonableness. Consider choice of the most appropriate model to be part of the verification process.

Many estimates involve more than a single step, one or more transformations, and sometimes fitting of non-linear regression. In these situations, make a single comparison of observed and estimated values of the dependent variable to produce an overall standard error of estimate. Where the same value is estimated by two or more methods, use these standard errors to help decide which is the preferred method.
35.2 - Location and Subsample Differences. Verify a system of equations by using them to prepare tables and graphs of the estimated values, and compare these with similar tables and graphs of average observed values. Carefully select class intervals for such comparisons. Use five to eight subdivisions of each range of sizes to get reasonable comparisons. Do not use too few subdivisions which may give poor definition of the estimated values and do not use too many which give poor definition of the observed values. Extend the estimates to one class above and below the observed range to check performance of the estimator outside of its intended range of applicability.
35.3 - Form Class and Merchantable Top. Use covariance analyses as the appropriate tests for form class and merchantable top. Test not only intercepts, but also coefficients of one or more independent variables. Use the tests to show which models are most sensitive to wild values (outliers). More robust models are preferable. Use these tests to determine the variation of trees within data sets from that among data sets, and thus, to forecast the total variation likely to be encountered when the estimator is used on an independent data set. This goes beyond the usual verification procedures. Use the results to avoid future concerns for bias.

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When data comes from two or more sources, use it as an opportunity to test differences among location or data source, as well as differences among trees within locations or data sets. Observed differences may indicate (1) variation among locations, (2) a biased estimator, or (3) ability of the worker. Evaluate differences with a broad view of the inevitable variation among trees, stands, localities, and mensurationists.
35.4 - Goodness of Fit. Fit the profile equation to subsets of the data; small, medium and large trees. Do this in addition to the covariance analyses given in section 35.3. Use three, four, or five sub-sets depending on both the amount of data and range of sizes. Recognize that significant independent variables that describe tree size for the entire data set often lose their significance or behave strangely because of the narrow range in the subset. Other independent variables may increase in significance. Fit equations to subsets while developing the model and later to verify it. See chapter 60 for procedures.
35.5 - Predictor of Standard Tree Results. Make estimates of several standard mensurational measures, such as form class, merchantable height, and so forth. Compare direct estimates made of these measures with similar measures derived from profile equations. Profile equations usually are fit with the square of diameter as the dependent value; therefore, those measures that involve averages of diameter may appear to be biased. If these apparent biases are small and in the right direction, consider the profile equation to be verified.
35.6 - Comparing Profile and Direct Estimates. Verify volume estimators derived as integrals of profile equations by comparing their volume with that of direct estimates. Direct comparison of two estimates may show that they are different in some respects. Expect these differences to be statistically significant, but to reveal nothing about which is the better estimate. In comparing two estimators to the data set on which they were based, expect the standard errors of estimate to be nearly the same, and neither estimator to show any great deviation of the "observed minus estimated volumes" when plotted over the variables entering the final estimating equation. Rework profile equations that cannot pass this verification test.

36 - PRIMARY PRODUCT ESTIMATORS. Recognize that the amount of primary product (lumber, plywood, pulpwood, and similar products) manufactured from a tree or a log is closely related to the volume it contains, but it is not the same thing. Expect estimates of primary product potential, especially of sawn lumber to be excessively complicated by "standards and definitions," by "adjustment factors," and by "allowances" peculiar to log scaling rules. If such estimators are necessary, use the procedures in the following sections.
36.1 - Sawn Lumber. Use the board foot as the unit of measure to be estimated. Recognize that a board foot is based upon the lumber piece's nominal dimensions and that under modern size standards and after finishing, dimensions are reduced from the nominal dimensions. For example, a piece of 2 x 4 measures 1.5 inches by 3.5 inches in thickness and width while length is a full measure.

Estimate sawtimber quantities by log rules and understand the many assumptions that underlie each rule. Recognize that most existing

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log rules are out of date and do not describe modern markets and milling practices well. In almost all cases, the log rules underestimate the quantity of lumber sawn from a log or tree by a modern sawmill. Identify the excess of production over prior estimate as overrun.

Use the Scribner Decimal C log rule, Cubic log rule, or the International 1/4-inch rule authorized at 36 CFR 223.3 to assess product amount in the tree segments. Follow procedures in FSH 2409.11, National Forest Log Scaling Handbook, or FSH 2409.11a, Cubic Scaling Handbook. If the lumber estimator is based on the production experience of a particular sawmill and a sample of trees, the estimator would be useful only for the mill for which it was based. Use either stem profile equation estimators or direct estimators.
36.11 - Stem Profile Equation Estimators. Ordinarily, use a stem profile equation to predict diameter inside bark at specified points on the tree bole. Secure or prepare a program, or preferably a subroutine, and use it to calculate the log rule volume. For any subject tree, use the following steps:

1. Pass tree measurements to the board foot subroutine.
2. Mathematically divide the tree stem into logs and segments according to direction in FSH 2409.11a.
3. With the stem profile equation, calculate a scaling diameter for each log (identified in preceding step 2).
4. Apply either the Cubic, Scribner Decimal C, or the International $1 / 4$-Inch rule to the scaling diameters and lengths to obtain a gross lumber scale for each log segment in the tree.
5. Sum the scale of individual $\log$ segments to obtain gross total scale for the whole tree.
6. Do not apply statistics to the results.

Use stem profile, when possible, since revised merchantability standards may be imposed on the calculation at any time. This method does not predict net volumes or defect, but it allows more accurate adjustment for defect than does a direct estimator.
36.12 - Direct Lumber Estimators. Use similar procedures in deriving direct estimators for board feet of lumber as for direct merchantable volume estimators (sec. 34). Fell, measure, and scale a sample of trees according to the appropriate scaling procedures and merchantability standards. Scale gross not net amounts. Adjust for defect by other means when the estimators are applied (FSH 2409.12). Construct a mathematical model to describe the relationship of lumber potential in the tree to tree measurements of dbh, height, and sometimes form. Height may be total height, merchantable height in feet or meters, or merchantable height in logs or logs and half logs. Use total height for conifers and for hardwoods with a predominantly central stem. In these trees, total height is almost as strongly correlated with lumber content as are merchantable height measures, and it is easier to measure accurately and unambiguously. In hardwoods with broad crowns, use merchantable heights. Use one of two basic models: (1) express board feet as a function of tree measurements, or (2) use the board foot/cubic foot ratio as the dependent variable.

Base direct estimators on a particular set of merchantability standards, and when merchantability changes, derive a different estimator. Avoid including merchantable top diameter as an independent variable.
36.12a - Board Foot Models. The general form of these models is:

$$
\mathrm{BF}=\mathrm{b}_{0}+\mathrm{b}_{1} \mathrm{D}^{2} \mathrm{H}+\mathrm{b}_{2} \mathrm{DH}+\mathrm{b}_{3} \mathrm{H}+\mathrm{p}(\mathrm{D}, \mathrm{H})
$$

where:
$p(D, H)=$ such polynomial and interaction terms in $D$ and $H$ as are necessary to obtain a satisfactory fit over the domain of $D$ and $H$.

If desirable, expand this type of model to include measures of form if any have been taken. Estimate parameters by ordinary least squares (OLS) regression. Board feet are related to volume, but the cubic dimension of this type of model is not accurate. The board foot/cubic foot ratio increases with tree size rather than remaining constant, and may lead to introduction of some higher power polynomial terms which detract from the estimator. Variance about this regression surface is not homogeneous, consequently, use observation weights proportional to $\left(D^{2} H\right)^{-n}$. Do not expect the optimal value of the exponent, $n$, to be the same as might be optimal for fitting direct volume estimators to the same sample of trees.
36.12 b - Board Foot/Cubic Foot Ratio Models. Because board feet is a poor dependent variable, it is often more practical to use the board foot/cubic foot ratio model:

$$
R_{b}=f(D, H, F)
$$

where:

$$
\begin{aligned}
\mathrm{R}_{\mathrm{b}}= & \mathrm{BF} / \mathrm{V}_{\mathrm{m}} \\
\mathrm{f}(\mathrm{D}, \mathrm{H}, \mathrm{~F})= & \text { A monotonically increasing function of } \mathrm{D}, \mathrm{H}, \text { and } \mathrm{F} \\
& (\mathrm{if} \text { a measure of form is available) asymptotic to } \\
& \mathrm{R}_{\mathrm{b}}{ }^{\star} \text { Where } \mathrm{R}_{\mathrm{b}} \text { is some board foot/cubic foot ratio } \\
& \text { which can never be exceeded no matter how large the } \\
& \text { tree. }
\end{aligned}
$$

A possible form for $f(D, H, F)$ is:

$$
\mathrm{R}_{\mathrm{b}}=\mathrm{b}_{0}-\mathrm{b}_{1} \mathrm{D}^{-1}+\mathrm{b}_{2} \mathrm{D}^{-2}+\mathrm{b}_{3} \mathrm{D}^{-3}
$$

where $b_{0}$ is $R_{b}{ }^{*}$ and all coefficient estimates are positive in absolute value. If the statistical estimate of $b_{0}$ is not satisfactory, determine $R_{b}{ }^{*}$ exogenously with the fitted model then being:

$$
\mathrm{R}_{\mathrm{b}}-\mathrm{R}_{\mathrm{b}}^{*}=-\mathrm{b}_{1} \mathrm{D}^{-1}-\mathrm{b}_{2} \mathrm{D}^{-2}-\mathrm{b}_{3} \mathrm{D}^{-3}
$$

Hypothesize that $\mathrm{R}_{\mathrm{b}}{ }^{*}$ increases with tree height, in which case the model becomes:

$$
R_{b}=\left(a_{0}+a_{1} H\right)-b_{1} D^{-1}-b_{2} D^{-2}-b_{3} D^{-3}
$$

If any of the coefficients $b_{i}$ seem to be related to tree height, substitute the appropriate function of $H$ for $b_{i}$. Regardless of how the model is expanded, maintain the property of being monotonically
increasing asymptotic to $\mathrm{R}_{\mathrm{b}}{ }^{*}$ (which may vary with H ). This may be more easily done with the model:

$$
\mathrm{R}_{\mathrm{b}}=\mathrm{R}_{\mathrm{b}}^{*}\left[1-\mathrm{b}_{1} \exp (-\mathrm{XB})\right]
$$

where:
$\mathbf{X B}=$ a general increasing linear function of $D$ and/or $H$ such that $\mathbf{X B} \geq 0$ for all $D$ and $H$.

For fitting this model, initially estimate $R_{b}{ }^{*}$ exogenously. If the asymptote is a function of $H$, determine the relationship exogenously. Then estimate the vector of coefficients, B, by fitting:

$$
\ln \left(1-R_{b} / R_{b}{ }^{*}\right)=\ln \left(b_{1}\right)-x B
$$

Do not regard the coefficients in $\mathbf{B}$ as having any particularly desirable statistical properties beyond $\exp (-\mathbf{X B})$ being a good transformation of $D$ and $H$ for describing the board foot/cubic foot ratio. Then fit:

$$
\mathrm{R}_{\mathrm{b}}=\mathrm{a}_{0}-\mathrm{a}_{1} \exp (-\mathbf{X B})
$$

where:

$$
\begin{aligned}
& \mathrm{a}_{0}=\mathrm{R}_{\mathrm{b}}^{*}, \text { and } \\
& \mathrm{a}_{1}=\mathrm{b}_{1} \mathrm{R}_{\mathrm{b}}^{*}
\end{aligned}
$$

If $R_{b}{ }^{*}$ is a function of $H$, then substitute that function for $a_{0}$ and $a_{1}$. If $a_{0}$ differs greatly from the initial estimate of $R_{b}{ }^{*}$, then repeat the process using $a_{0}$ as the initial estimate, re-estimate $\mathbf{B}$, then finally re-estimate $a_{0}$. A better alternative to this procedure is to use nonlinear regression software to estimate all coefficients including B.

Variance around this regression surface will most likely be nearly homoscedastic, so that observation weights are all unity.

To construct a board foot estimator using merchantable top diameter as an independent variable, scale trees to a variety of merchantable tops. To avoid correlated regression errors, scale each tree to only one merchantable top and cover a range of top diameters by selecting a large enough sample of trees. If this is not practical, scale the same trees to a variety of merchantable tops, and accept the effects of correlated errors. Fit estimators based on the same model separately for each different merchantable top. Examine and model the relationships between coefficient estimates and merchantable top. Substitute the resulting functional relationships between coefficients and merchantable top into the $R_{b}$ model and estimate all coefficients simultaneously. Recognize that this process is not easy, that satisfactory results are elusive, and the result can be attained with better accuracy using stem profile equations.
36.13 - Estimation of Losses in Lumber Potential. Recognize that lumber potential of a tree may be severely reduced by volume loss, and also by frost cracks, ring shake, end checking, or other cracks that reduce lumber potential without volume loss.

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Follow procedures for volume loss adjustment in scaling given in FSH 2409.11a or FSH 2409.11. Develop the information from cut logs. However, recognize that the estimates associated with whole-tree lumber potential estimators rely upon information taken by the user on standing trees as given in FSH 2409.12. Generally:

1. Identify the portion of the tree subject to the damaging agent,
2. Estimate the fraction of that portion of the stem which has lost its usefulness.

For example, common cruising deductions such as "30 percent of the butt log," or "half the second log" are applications of this procedure. Detailed instruction for field application are given in FSH 2409.12 and Region or Forest supplements to it.

To use stem profile equations to calculate scaling diameters along the tree bole, only record the lower and upper limits of the affected portion of the stem and the proportion of that stem segment which has lost its lumber potential. Divide the remaining unaffected stem into logs according to existing procedures and scale by applying the selected scale rule to calculated diameters.

With direct lumber potential estimators, use an approach similar to that outlined for stem profile equations (section 32). Estimate the average proportion of the tree's potential lumber log by log then estimate the defect in each log and obtain estimate of the proportion of potential lumber lost in the whole tree. Use shortcuts such as the table shown in FSH 2409.12, section 22.31 a to make the process more manageable.
36.2 - Veneer. Estimate veneer in units of a square foot $3 / 8$ inch thick. For example: a 4 feet by 8 feet sheet of $3 / 4$ inch thick plywood contains $4 \times 8 \mathrm{X}(3 / 4) /(3 / 8)=64$ square feet. A 4 feet by 8 feet sheet of $1 / 4$ inch thick plywood contains $4 \times 8 \mathrm{X}(1 / 4) /(3 / 8)=$ 21.3 square feet

When preparing the estimator, recognize that veneer potential is closely related to that part of the volume contained in the veneer bolt, and veneer is turned from bolts that are nominally 8 feet long. Veneer yield also differs with species, age, and condition of the timber. Estimate utilization factors from a mill production study. Exclude the volume not used for veneer for the following reasons:

1. The length of bolts contains a trim allowance and veneer from that portion of the bolt is usually scrap. Therefore, use a locally established trim allowance when dividing a tree stem into veneer bolts.
2. A core is gripped by the veneer lathe chucks and is not used for veneer. Identify the core size and do not include it in the veneer potential.
3. Sheets of veneer come from a cylinder of wood contained within the bolt with the diameter equal to the small end. Due to taper and surface irregularities, the first veneer turned from a fresh bolt comes from it in pieces (fishtails) of gradually increasing size. Some pieces are used and some are wasted. Exclude the unused portion.
4. A small part of the volume in the cylinder is also unused because it contains knots or other defects. Exclude the unused portion.

To derive an estimate of veneer potential, begin with the following notation:

```
V
V
V
Vf}=\mathrm{ volume in the "fishtail" portion of the bolt
d
ds}= diameter at small end of the bol
d
l = nominal length of the bolt
k = pi/(4*144) = 0.005454154
P = expected veneer yield of the bolt
```

Then:

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{t}}=\mathrm{kl}\left(\mathrm{~d}_{\mathrm{l}}{ }^{2}+\mathrm{d}_{\mathrm{s}}{ }^{2}\right) / 2 \\
& \mathrm{~V}_{\mathrm{r}}=\mathrm{k} \mathrm{~d}_{\mathrm{s}}{ }^{2} \mathrm{l} \\
& \mathrm{~V}_{\mathrm{c}}=\mathrm{k} \mathrm{~d}_{\mathrm{c}}{ }^{2} \mathrm{l}
\end{aligned}
$$

and:

$$
\begin{aligned}
\mathrm{V}_{\mathrm{f}} & =\mathrm{V}_{\mathrm{t}}-\mathrm{V}_{\mathrm{r}} \\
& =\mathrm{k} * 1 *\left[\left(\mathrm{~d}_{1}^{2}+\mathrm{d}_{\mathrm{s}}^{2}\right) / 2-\mathrm{d}_{\mathrm{s}}{ }^{2}\right] \\
& =\mathrm{k} * 1 *\left(\mathrm{~d}_{1}^{2}-\mathrm{d}_{\mathrm{s}}^{2}\right) / 2
\end{aligned}
$$

There are 32 square feet of $3 / 8$ inch thick veneer per cubic foot of volume. If $U_{f}$ represents the utilization factor for fishtail material and $U_{r}$ the utilization factor for cylinder material, then

$$
\begin{aligned}
P & =32 \mathrm{U}_{\mathrm{f}} \mathrm{~V}_{\mathrm{f}}+32 \mathrm{U}_{\mathrm{r}}\left(\mathrm{~V}_{\mathrm{r}}-\mathrm{V}_{\mathrm{c}}\right) \\
& =32 \mathrm{U}_{\mathrm{f}} \star \mathrm{k} * 1 *\left(\mathrm{~d}_{\mathrm{l}}{ }^{2}-\mathrm{d}_{\mathrm{s}}^{2}\right) / 2+32 \mathrm{U}_{\mathrm{r}} \star \mathrm{k} * 1 *\left(\mathrm{~d}_{\mathrm{s}}{ }^{2}-\mathrm{d}_{\mathrm{c}}{ }^{2}\right)
\end{aligned}
$$

The utilization factors $U_{f}$ and $U_{r}$ differ from one veneer plant to another. The factors are linear in the equation. Assume a nominal bolt length of eight feet, and simplify the equation to:

$$
\mathrm{P}=0.69813 \mathrm{U}_{\mathrm{f}}\left(\mathrm{~d}_{1}{ }^{2}-\mathrm{d}_{\mathrm{s}}{ }^{2}\right)+1.39626 \mathrm{U}_{\mathrm{r}}\left(\mathrm{~d}_{\mathrm{s}}{ }^{2}-\mathrm{d}_{\mathrm{c}}{ }^{2}\right)
$$

Apply this equation to bolts, and to only bolts with $d_{s} \geq d_{c}$. Estimate the appropriate utilization factors and use a stem profile equation to divide the predicted stem into bolts. Estimate the diameters $d_{1}$ and $d_{s}$ for each bolt and calculate its potential veneer
outturn. Obtain total expected veneer outturn for the tree by summing up bolts.

If a direct estimator is desired, measure a sample of felled trees at the points where the trees would be bucked for bolt production. Calculate veneer content for the bolts that would be obtained thereby and sum over the whole merchantable stem. This constitutes an observation on the dependent variable, veneer content. Estimate it by a regression equation similar in form to a merchantable volume equation.
36.3 - Poles and Pilings. Utility poles and pilings are usually measured by length and sold by length. They are usually further classified by grades which are determined partly, though not entirely, by diameter. Determine the measurement and grading of poles and pilings in use locally. For example, specifications for piling in the Puget Sound area are different from specifications on the Mississippi Gulf Coast or the Atlantic Coast. Due to variation in standards from one geographic area to another, no standard methods are specified.
36.4 - Fiber Products. This category of products includes paper, pressed fiberboard or fiberboard products, fuel, or any manufactured product in which wood is reduced to chip or fiber form. The quantities of such products which can be produced are closely related to the volume of wood. Derive conversion coefficients relating to pounds of paper or square feet of chipboard to the volume of wood going into the production process and any waste from observation of the process. Use elementary statistical estimation techniques to determine product outturn for a given cubic unit of wood volume.

Use the procedures in the Cubic Scaling Handbook (FSH 2409.11a) to predict fiber content of logs. From the many defects described therein, deduct only for rots, voids and char. Use the cubic volume to establish relationships with other predictors such as weight.

Because it is often easier to weigh a quantity of wood on trucks than to measure its volume, weight may be used as a predictor variable for fiber products rather than volume. Ensure that variation deriving from changes in weight from such things as the fuel that is in the tank, mud stuck to the mud flaps and undercarriage, the tire chains and binder chains aboard, drivers, ice, snow, and so forth is accounted for. Include a method to account for the variable moisture content of the wood. For example, if wood remains for a long time in a deck and dries out before being weighed, the amount of product per pound will be greater than for freshly cut wood. Density of wood also varies by species. Prepare and use different weight-to-product conversion coefficients for each species or for groups of species with similar characteristics. If needed, do likewise for live (high moisture) and dead (low moisture) within a species or group. Establish a procedure to predict fiber content of logs when loads are of mixed characteristics.

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TIMBER VOLUME ESTIMATOR HANDBOOK

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CHAPTER 40 - VALIDATION AND CALIBRATION

This Chapter contains direction to determine how well the proposed equation predicts the variable of interest, usually either bole diameter or volume, and if a sufficient degree of accuracy has been achieved.

41 - BASICS OF VALIDATION AND CALIBRATION. Use validation to determine the utility of a proposed estimator for a general population of trees (target population) to which the equation is applied. Use calibration to adjust a volume estimator for a specific or local target population that has been shown to vary from the general target population. Consult the intended users of an equation to determine the degree of accuracy the equation must achieve, and specify it before beginning the validation or testing phase of equation development.
41.1 - Species. Develop a profile or volume equation for a single species. However, recognize that a single-species equation sometimes must be used for another species or for a species group. Evaluate and group similar species prior to the data gathering process. Specify a species or group so that the equation user neither needs to determine the range of species for which the equation applies nor to validate the results.
41.2 - Diameter Breast Height and Height. Ensure that the ranges of diameter breast height (dbh) and total height of the sample span the ranges of dbh and total height of the target population. If trees in the target population are either smaller or larger than those making up the estimation data set, determine the accuracy of the extrapolation.

Divide the trees into diameter classes and evaluate the equation across the range of tree sizes in the target population. Make the class sizes as small or large as needed to determine whether the equation works for both small trees and large trees. Determine the range of diameters for which the equation is to be used and divide it into four to six diameter groups. For example, if candidate trees range from 6 to 36 inches dbh, they may be divided into five groups as follows: 6-12 inches, 13-18 inches, 19-24 inches, 25-30 inches, and 31-36 inches. Use enough classes to detect estimation bias or inaccuracies that are attributable to tree dbh.

Divide the trees into total height classes as was done with the dbh classes and determine if the equation works for both the shortest and tallest trees in the population. Use four to six height classes, spanning the entire range of population heights.
41.3 - Volume. Use accuracy of tree volume prediction as the criterion to judge a candidate equation. Determine the prediction variable of interest which may be volume of the total tree or any merchantable portion of the tree such as individual logs, the total sawlog volume, or tree volume to the pulpwood top.

Specify, in advance, a single prediction variable to evaluate. Avoid evaluating an equation for multiple accuracy criteria which may cause confusion. For example, an equation may give excellent results for predicting total tree volume but give slightly biased results for the butt $\log$ or for the tip of the tree. If so, use total tree volume as

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the most reliable volume estimate to evaluate, even though the sawlog portion of the tree may be the real variable of interest. If the total volume estimate is reliable, expect the sawlog volume estimate to be reliable, because a high proportion of the total tree volume occurs in the sawlog portion of the tree.
41.4 - Geographic Differences. When a species occurs across several geographic or physiographic areas, use tree form as the indicator of tree differences. If two trees of equal dbh and total height grow on two distinctly different physiographic areas, expect measurable differences due to different shape and form. Eliminate some of the differences due to location by including a measure of form in the profile or volume model.

Without an available measure of form in the estimator, develop separate volume functions based on geographic location.

42 - VALIDATION PROCESS. Use procedures to validate an equation that range from a visual comparison of old and new volume tables to rigorous statistical tests for determining measurable statistical differences. Consider no single test or procedure sufficient for all equation validations. Ensure that validation consists of enough statistical tests and comparisons to determine the accuracy and usefulness of the candidate equation for the target population.
42.1 - Measurements. Take measurements on each tree identical to those required to use the equation plus those necessary to validate the accuracy of the equation. For example, to validate a candidate equation for cubic foot volume to a 4-inch top diameter outside bark, measure dbh, diameter at the upper merchantability point (Du), and the height where the diameter is 4 inches inside bark (H4) for individual trees. Determine cubic foot volume of the test trees by taking additional measurements along the bole at fixed height intervals and calculate volumes for each segment using standard volume formulae. Reference and use a standard geometric formulae from a forest mensuration textbook.

Consider each segment of the tree to be a truncated geometric solid. Calculate volumes in the bottom 10 percent of the tree using the formula for a truncated neiloid. Calculate volumes in the top 10 percent of the tree using the formula for a truncated cone. Calculate volumes in the middle portion of the bole using the formula for a truncated paraboloid. Calculate the volume of each bole segment. Sum the segments to obtain the measured tree volume from stump to a 4 inch diameter inside bark (V4).
42.2 - Statistical Tests. Within each group, calculate the statistics needed to help evaluate the accuracy and precision of the equation. Use all available statistics since no single statistic is adequate. See section 42.3 for an example application. In addition to statistics, plot the individual residual values against the variables used in the equation (section 42.29).

Calculate the following statistics for each group:

1. Sample mean, YBAR.
2. Mean bias, BBAR.
3. Percent bias, PBIAS.
```
    4. Coefficient of determination; R-Squared or Fit Index (FI).
    5. Standard error of estimate, SE.
    6. Coefficient of Variation, CV.
    7. 95% Chi Square Error Limit, CSEL.
42.21 - Variable Descriptions. The following variables are used
throughout the remaining sections of this chapter:
1. N - The number of trees or observations in the group.
2. VCAP - The predicted volume of an individual tree.
3. VOBS - The observed or measured volume for a tree.
4. RESID - The residual for a tree prediction;
    RESID = VCAP - VOBS.
5. TSS - Total sum of squares for the observed volumes;
    TSS = sum(VOBS*VOBS), summed over the N values.
6. RSS - Residual sum of squares for the group;
    RSS = sum(RESID*RESID), summed over the N values.
42.22 - Sample Mean. Calculate the sample mean as the average measured tree volume for the group. Use it to give an indication of the relative size of the measured trees and to predict other statistics.
\[
Y B A R=\frac{\sum^{N} V O B S}{N}
\]
```

42.23 - Mean Bias. Calculate the mean bias as the average of the difference between measured volume and calculated volume for all trees in the group. The individual tree differences are the prediction residuals. Use these residuals to calculate most evaluation statistics. Expect that the more precise and accurate the equation, the smaller the individual residual, the smaller the sum of the residuals, and the smaller the mean bias. If the equation could predict each volume exactly, the value of the mean bias would be zero. Thus, a smaller mean bias provides greater confidence that the equation is producing accurate results.

$$
B B A R=\frac{\sum^{N} R E S I D}{N}
$$

42.24 - Percent Bias. Express percent bias as the ratio of mean bias to sample mean in percent. Use this as a measure of how far the average prediction misses the average "true" or measured value. This statistic should be "small," with small being as close to zero as possible and no larger than can be tolerated. Consider values smaller than 10 percent to be acceptable.

```
PBIAS = 100(BBAR/YBAR)
```

42.25 - Coefficient of Determination (R-Squared or Pseudo R-Squared). Use $R$-Squared as a measure of how well the equation predicts individual volumes. The magnitude of $R$-Squared lies between 0.0 and 1.0. If the equation exactly predicts the volume of every tree, then each residual is zero, the RSS is zero, and R-Squared $=1.0$. The R-Squared should be as close to 1.0 as possible.

Use pseudo R-Squared or Fit Index (FI) in place of $R$-Squared when an equation is fit using techniques that produce statistically biased predictions. Interpret Fit Index the same as R-Squared and it should have magnitude between 0.0 and 1.0 . However, recognize that if the equation fit is extremely poor, FI may sometimes be negative, since FI is computed using raw, not mean-corrected, squares.

R-Squared $=[(T S S-R S S) / T S S]$
42.26 - Standard Error of Estimate. Use the standard error of the estimate as a measure of the variation of the observed volumes not accounted for by the equation. This statistic should be "small" relative to YBAR.
$S E=[R S S /(N-p)] * * 0.5$,
where:
$p$ is the number of coefficients in the taper equation.
42.27 - Coefficient of Variation. Use the coefficient of variation as the measure of the relative size of the standard error to the group mean. If the value is 0.0 , then no prediction residuals exist and the equation exactly predicts every volume. This value should be as close to 0.0 as possible. Consider a value of 10 percent or less to be acceptable.

```
CV = 100(SE/YBAR)
```

42.28 - Chi Square Error Limit (CSEL). Consider this statistic to be a modification of Freese's Chi Square statistic (Freese, Frank. 1960). This may be a more meaningful accuracy statistic than Freese's. For example, if CSEL $=4.8$ percent, expect 95 percent of the deviations to be within +- 4.8 percent of their estimated values.

```
CSEL = 100{Z*Z[sum[(RESID/VCAP)*(RESID/VCAP)]/CHISQ]**0.5}
```

where:
$Z=1.96$, the value of the standard normal deviate at the 95\% probability level,

CHISQ $=0.853+v+1.645(2 v-1) * * 0.5$,
$\mathrm{v}=$ degrees of freedom for the Chi Square statistic; $v=$ $\mathrm{N}-1$.
42.29 - Plotting of Residuals. Plot the residuals (observed predicted) against the variable used in the estimation process, as well as other variables such as crown class, location, and elevation. Make special note of plotting the predicted merchantable height verses the actual merchantable height.

Expect no correlation between the values of the input variables and the residuals. The plotted residuals should form a horizontal band spanning the range of the equation variable. If the predicted volumes are plotted against the measured volumes, the plotted data should generally show a straight line trend at a 45-degree angle with most of the points clustered near the line.
42.3 - Example - Short Leaf Pine In Alabama. A form class profile equation was developed for shortleaf pine in Alabama. It was conditioned so the plotted taper line would pass through the points $(4.5, D),(17.3, D u)$, and $(H 4,4.0)$; that is, when the measured height on the bole is 4.5 feet, the equation will equal the measured dbh, at the Girard Form Class height of 17.3 feet, the equation equals the measured outside bark form diameter (Du), and at the height of the merchantable bole to a 4-inch top (H4), the outside bark bole diameter is 4 inches.

To validate this equation, evaluate the equation for a sample of trees representative of short leaf pine in Alabama where the equation is expected to be applied. Establish the minimum and maximum size trees for which the equation is to be valid. Measure representative trees to establish a data set for validation testing.

In this example, a representative sample of shortleaf pine trees were selected and measured in conjunction with existing logging
operations. In addition to the measures of dbh, Du, and $H 4$, required for equation use, bole diameter outside bark and bark thickness were measured at 5-foot intervals on the bole starting with a 1-foot stump. Volume of each tree was computed using formulae for geometric solids. The computed statistics (section 42.2) are presented in exhibit 01.

The equation was compared for three diameter groups, three height groups, and for all groups combined. The mean volume of all trees in the first diameter group (6-10 inch class) was 17.60 cubic feet. The mean bias shows the equation slightly underestimated tree volume by an average of -0.483 cubic feet or a percent bias of -2.745 . The fit index is 0.933, which was reasonably close to 1.0 , indicating that individual predictions were close to the measured volumes. The standard error of 1.04 compared to the mean of 17.60 gave a good coefficient of variation of 5.912 percent. The chi square error

```
limit showed that 95 percent of the predictions are within 6.58
percent of their true value for that diameter group.
A scan of the rest of exhibit 01 showed all FI's larger than 0.90,
coefficients of variation near 5 percent, and chi square error limits
ranging from 6.2 to 8.2 percent. The tallest height group showed
slight differences from those just described. These statistics were
acceptable and were caused by the relatively few number of trees
represented in that group.
The evaluation across all groups showed a mean bias of -1.001
compared to a mean of 39.91, indicating a slight underestimation of
volume by -2.509 percent. The FI of 0.988 compared very well to the
possible 1.0. The overall chi square error limit showed that 95
percent of the volume predictions were within 8.147 percent of their
measured values.
Based on these statistics, the equation was found to be useful for
volume prediction with little or no adjustment. If adjustment is
needed, use calibration procedures in section 43.
```

42.3 - Exhibit 01

Example Table of Statistics for Validation of Form Class Taper Equation
in Alabama Shortleaf Pine.

| Statistic | Diameter Class Group (inches) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 6-10 | $11-15$ | 16-20 | $21-25$ |
|  |  |  |  |  |  |
| Mean volume, cuft | 17.60 | 34.39 | 65.22 | * |  |
| Mean bias, cuft |  | -0.483 | -0.793 | -1.794 | * |
| Percent bias | -2.745 | -2.306 | -2.751 | * |  |
| Fit index | 0.933 | 0.969 | 0.932 | * |  |
| Standard error, cuft | 1.040 | 1.903 | 3.340 | * |  |
| Coef. of variation | 5.912 | 5.533 | 5.121 | * |  |
| Chi sq. error limit (\%) |  | 6.580 | 8.210 | 7.140 | * |

$-$

|  | Bole Height to 4-inch Top dob (feet) |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  | $<25-26-50$ | $51-75$ | $76+$ |  |  |



| Statistic | Across All Groups |  |
| :--- | :---: | :---: |
| Mean volume, cuft | 39.91 |  |
| Mean bias, cuft | -1.001 |  |
| Percent bias | -2.509 |  |
| Fit index | 0.998 |  |
| Standard error, cuft | 2.182 |  |
| Coef. of variation | 5.467 |  |
| Chi sq. error limit (\%) |  | 8.147 |

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43 - CALIBRATION. This procedure is the final step in the validation process and may be used to develop a correction factor which becomes part of the final estimation process. Use calibration to eliminate the bias found between the estimator and the local condition; however, do not use it to create a new or modified volume estimator for use elsewhere.
43.1 - When to Calibrate. Determine the need to adjust a volume equation to local conditions subjectively, depending on the importance of its use. Recognize that obtaining an accuracy better than 2.5 percent is unlikely and that accuracy of 10 percent or more is inaccurate and should be reevaluated (FSH 2409.12, sec. 22.23).

To determine if a profile/volume equation needs refinement, estimate cubic volume using the model on the target population. Calculate the actual volume of the target population by means of a local sample of limited size measured in great detail. Since the need for calibration is a subjective judgment, it may not be necessary to calibrate to every local condition but simply to realize the strengths and weaknesses of the estimates. However, when predicted volumes versus actual volumes have a difference of 10 percent or more, consider using a different model, developing a new model, or calibrating the current model. If it is decided to calibrate, prepare a graphical solution of predicted over actual volume along with the validation statistics.
43.2 - How to Calibrate. Use information readily available from the validation statistics to indicate bias. Use graphical presentation of the predicted volume over the actual volume to further support the general trend of the bias across the range of the target population.

Expand on the statistics and graphical solutions by using two common techniques.

1. Adjust the intercept of the profile equation (often referred to as a percentage adjustment), if it has one. This procedure may be acceptable if the bias is relatively constant over the range of the local volume estimates and a single coefficient would be adequate to correct for the volume difference (bias).
2. Adjust each diameter class by an appropriate correction factor. Develop a functional relationship of volume difference over diameter, and use this method if the volume difference is correlated to the changes in diameter.
43.3 - Field Procedures. Regions may supplement this section with procedures for calibration of equations in local use.

44 - CONCLUSION AND DOCUMENTATION. Document validation and calibration projects in a consistent format using an outline similar to that displayed in exhibit 01 . Include a statistical summary similar to that shown in section 42.3 exhibit 01. Include graphical comparisons, if any are made, and a concise narrative explanation of the validation/calibration project results. Distribute informational copies to the affected National Forests and the national data base coordinator (sec. 04.13). If calibration of volume estimators is expected to be necessary, provide direction for field application in a regional supplement to section 43.3.

44 - Exhibit 01

Validation/Calibration Project Report Outline
I. Introduction: A brief introduction to the project, including a general description, objectives, purpose, and the need for the project. If a project plan was prepared, include it by reference, and enclose a copy.
II. Equations Being Tested: List the equations being tested and their geographic limitations (Region-wide, forest by forest, and so forth).
III. Results.
A. Data Summary.

1. Data items gathered.
2. Range and averages of data by species and equation.
a. DBH.
b. Height.
c. Predicted Volume/Tree.
d. Actual Volume/Tree.
e. Other Pertinent Data Items.
B. Data Analysis: Predicted versus Actual Values - summary of comparisons by equation.
C. Statistical Variation and Tolerance Limits: Are
predicted versus actual variations acceptable? Describe tolerance limits.
D. Limits of Validation Results.
3. Region-wide.
4. Forest by forest.
IV. Need for Calibration: Does the data analysis indicate a need for calibration? If so, are more data needed, or will validation data be used? If calibration is necessary, include the following:
A. Calculation of ratio or coefficient adjustment.
B. Limits of calibration: Forest by forest or Region-wide.
C. Regional direction: Include Regional direction on use of calibration (section 43.3).
V. Data Storage: Explain where data are stored, how to access data and the electronic and hardcopy format.

CHAPTER 50 - APPLICATION

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CHAPTER 50 - APPLICATION
50.2 - Objective. Use volume estimates to:

1. Predict current standing timber inventory volumes, future potential yields, and long term sustained yield.
2. Develop timber sale volumes based on timber cruise data.
3. Analyze efficiency and effectiveness of timber management practices.

51 - STANDARDIZATION.
51.1 - Approved Regional Volume Estimators. Regional Foresters should supplement this section with the approved regional and local volume estimators for all tree species and tree conditions found within the region (sec. 04.21). Include tree quality or value predictors. Display volume functions by species and geographic limitations. Describe any other limitations on volume estimator use. If the volume estimator has been calibrated, include the calibration functions.
51.2 - Total Cubic Content. The volume of the tree is its total cubic content. Do not confuse the total cubic content with the cubic product content of the tree as determined by a scaling process and formula application.
51.21 - Smalian's Formula. Use Smalian's formula, which is the form of a frustum of a paraboloid to estimate the cubic volume in a log. A discussion of this formula is contained in Bruce and Schumacher (sec. 08). It is the basis for the cubic log scaling rule (FSH 2409.11a).

Smalian's formula is based on equation:

$$
\mathrm{V}=\mathrm{K}\left(\mathrm{D}_{0}^{2}+\mathrm{D}_{1}^{2}\right) \mathrm{L}
$$

where,
V = volume in cubic feet
$D_{0}=$ diameter at the small end in inches
$D_{1}=$ diameter at the large end in inches
$\mathrm{L}=$ length in feet
$K=.002727$
51.22 - Approximating Total Volume. When possible, mathematically integrate stem profile functions to obtain a volume function. There are some stem profile equations which do not have an integral form. In such cases, approximate a true integral by segmenting the tree into short pieces, calculating the volume for each piece, and adding up the pieces. Since the utility of most stem profile equations is best if they are imbedded into a computer program, consider the approximation procedure to be practical.

Use the following procedure to find the total fiber content of the tree:

1. Ground to 1 foot. Treat this segment as a cylinder with both large and small end diameter equal to the inside bark diameter calculated at a height of 1 foot.
2. One foot to the tip. Determine a diameter for each 4-foot piece up the tree starting at 1 foot, resulting in diameters at 1 foot, 5 feet, 9 feet, and so forth. Calculate the volume for each 4 -foot piece using Smalian's formula. Expect the top piece to be some length other than 4 feet and sum the pieces and the 1 -foot stump to equal the height of the tree. Section 51.31, Exhibit 01 illustrates this process.
51.3 - Merchantable Volume.
51.31 - Segmenting the Tree. Recognize that segment lengths affect both the calculation of cubic volume and board foot product estimation. See FSH 2409.12, Timber Cruising Handbook; FSH 2409.11, National Forest Log Scaling Handbook; and FSH 2409.11a, Cubic Scaling Handbook for additional direction. Use the following segmentation rules to determine the volume of a tree:
3. The 20-foot rule (FSH 2409.11, The National Forest Log Scaling Handbook).
4. The 16-foot rule (FSH 2409.11, The National Forest Log Scaling Handbook).
5. The 40-foot West Coast Bureau rule (FSH 2409.11, The National Forest Log Scaling Handbook).
6. The 20-foot cubic rule (FSH 2409.11a, The National Forest Cubic Scaling Handbook).
7. The nominal log length rule.

The first four rules are scaling rules and define how a log should be segmented if it were presented for scaling. For tree volumes in this handbook, it is assumed that the logs are uncut and presented for scaling in tree lengths.

If the nominal log length rule is used in cruising timber, it is customary to go up the tree in 16 foot increments until the stem diameter is less than specified for merchantability. The top piece may be less than the nominal 16 feet, and if so, round down to the nearest length that is a multiple of 2 feet. Consider visualized segments that are graded and have defect estimates made for them.
51.31 - Exhibit 01

```
Main Stem Total Fiber Content in Cubic Feet
```

| Heigh 100.0 | Diameter insid Bark (in.) 0.0 | Cubic Foo Volume |
| :---: | :---: | :---: |
| 97.0 | 2.66 | . 06 |
| 93.0 | 4.23 | . 27 |
| 89.0 | 5.5 | . 52 |
| 85.0 | 6.65 | . 81 |
| 81.0 | 7.73 | 1.13 |
| 77.0 | 8.78 | 1.49 |
| 73.0 | 9.78 | 1.88 |
| 69.0 | 10.77 | 2.30 |
| 65.0 | 11.74 | 2.77 |
| 61.0 | 12.67 | 3.25 |
| 57.0 | 13.55 | 3.75 |
| 53.0 | 14.38 | 4.26 |
| 49.0 | 15.17 | 4.77 |
| 45.0 | 15.93 | 5.28 |
| 41.0 | 16.66 | 5.80 |
| 37.0 | 17.37 | 6.32 |
| 33.0 | 18.05 | 6.85 |
| 29.0 | 18.72 | 7.38 |
| 25.0 | 19.36 | 7.91 |
| 21.0 | 19.99 | 8.45 |
| 17.0 | 20.61 | 8.99 |
| 13.0 | 21.21 | 9.54 |
| 9.0 | 21.8 | 10.09 |
| 5.0 | 22.38 | 10.65 |
| Ground FT. | 29.23 | 14.78 4.66 |
|  |  | $\overline{133.96}$ |

TIMBER VOLUME ESTIMATOR HANDBOOK
51.32 - Trim. In the rules discussed in section 51.31 there is an assumed trim. This is specified in advance and is usually . 5 feet per log or 4 inches per log. When merchantable volume is calculated, do not include trim. When selling sawlogs or veneer logs, do not charge for the trim piece.
51.33 - Rounding Diameters. When determining the merchantable volume, do not use log diameters in fractional form. For example, round a 12.3 inch diameter inside bark (dib) to 12 inches, a 12.6 inch dib to 13 inches.
51.34 - Top Log. Under the log scaling rules identified in section 51.31, segment the top log of a tree according to a published table, such as those found in the National Log Scaling Handbook, FSH 2409.11 (sec. 17), or calculate it according to the rule being used. Also, determine the merchantability (utilization) specification for top diameter.

As an example, a tree may be 54 feet in length from the stump to the minimum merchantable top diameter. Under the $20-f o o t$ rule no segment may be longer than 20 feet and should be in 2 -foot multiples. Thus the 54 -foot piece, may have three logs for which there would be 1.5 (0.5 x 3) feet of trim. Round the trim to 2 feet, and determine the "scaled" length to be 52 feet (54 minus 2). Divide the 52 -foot piece into segments of approximately equal length with the smallest log on top resulting in two logs of 18 feet and a top log of 16 feet.
51.35 - Merchantable Cubic Volume. After the tree is segmented, calculate cubic volume. Calculate the inside bark diameters using diameter breast height of the tree along with the height up the bole. Calculate height up the bole by adding the segments plus the trim allowance. For the butt log, the big end diameter is the inside bark diameter 4 feet above the large end. Thus, each log has a small end diameter, a large end diameter, and a nominal length. Use Smalian's formula to calculate the gross cubic volume. See FSH 2409.11a, Cubic Scaling Handbook for detailed direction on calculating merchantable cubic volume of each segment.

Recognize that Smalian's formula overestimates cubic volume for long segments and accordingly, avoid very long segments. Watch for cases where the merchantable cubic volume exceeds the total cubic volume calculated using the pseudo-integration method. Note also that in the scaling rules, diameters and lengths are rounded.
51.36 - Board Foot Scale. Use board foot log rules to estimate the products in board feet for logs of specific diameters and lengths. For rules such as Scribner Decimal C based on log diagrams, look up the Scribner Decimal C volume in tables in the National Forest Log Scaling Handbook, FSH 2409.11 or other factor tables. Recognize that factor tables may be slightly different from the actual Scribner table, especially for small diameter logs. Apply the board foot rule to the same segments which were calculated for the merchantable cubic volume.

For formula rules such as the International $1 / 4$ inch rule use the equation:

```
International 1/4 inch rule (Bd. ft.) =
0.049762 LD ' + 0.006220 L'D - 0.185476 LD + 0.000259 L'
-0.011592 L' + + 0.042222 L
```

where:

$$
\begin{aligned}
& \mathrm{D}=\text { diameter inside bark at the small end of the log in } \\
& \mathrm{L}=\mathrm{log} \text { lenes } \\
& \text { length in feet }
\end{aligned}
$$

Recognize that conversion between board foot rules and from board foot to cubic varies with log size and if conversion is necessary, convert on a log-by-log basis. Also board foot rules estimate lumber cut from logs, but actual output may be different. Calculate overrun to express the variation between the estimate (log scale) and the actual lumber (mill tally) sawn from the log:
percent overrun $=$ [(mill tally / log scale)]100
51.37 - Examples of Segmentation and Volume Calculation. See exhibits 01 through 06 which illustrate the segmentation rules described in 51.31 to 51.34 needed to calculate volume and product estimates. Exhibit 07 summarizes the volumes presented in the figures. Note that each method gives a slightly different answer. For the example tree, the differences are small. For other tree sizes, this difference could be large. Recognize that as the trees get shorter, the relative effect of the segmentation rule become more important.

```
TIMBER VOLUME ESTIMATOR HANDBOOK
51.37 - Exhibit 01
Segmentation According to the 16 Foot Rule (FSH 2409.11)
```



```
TIMBER VOLUME ESTIMATOR HANDBOOK
51.37 - Exhibit 02
Segmentation According to the 20 Foot Rule (FSH 2409.11)
```



TIMBER VOLUME ESTIMATOR HANDBOOK
51.37 - Exhibit 03

Nominal Log Length of 16 Feet. Top Segment Scaled as it is.


### 51.37 - Exhibit 04

Nominal Log Length of 16 Feet. 16 Foot Maximum Length Rule.


TIMBER VOLUME ESTIMATOR HANDBOOK
51.37 - Exhibit 05

Nominal Log Length of 16 Feet. If Top is Less than Half Log, use 16 Foot Maximum Length if More, Scale as it is.

| Height Above GroundFT) | Diameter Inside Bark rounded to nearest inch |  | Cubic Foot Volume | Board Foot Volume Scribner Int'nl 1/4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 87.0 | 6" dib 6.0 |  |  |  |  |
| $86.0 \xrightarrow{\longrightarrow}$ |  |  |  |  |  |
|  |  |  | 2.6 | . 5 | 1.0 |
| 77.5 | 9.0 |  |  |  |  |
|  |  |  | 5.5 | 3.0 | 3.0 |
| 67.0 | 11.0 |  |  |  |  |
|  |  |  | 15.1 | 7.0 | 8.0 |
| 50.5 | 15.0 |  |  |  |  |
|  |  |  | 24.0 | 14.0 | 16.0 |
| $34.0 \longrightarrow$ | 18.0 |  |  |  |  |
|  |  |  | 31.6 | 21.0 | 23.0 |
| 17.5 | 20.0 |  |  |  |  |
|  |  |  | 40.5 | 28.0 | 29.0 |
| Ground 1 $\qquad$ |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| (Trim length of .5 ft ) Total Volume |  |  | 119.3 | 73.5 | 80.0 |

## TIMBER VOLUME ESTIMATOR HANDBOOK

### 51.37 - Exhibit 06

Nominal Log Length of 16 Feet. Top Segment Rounded to Nearest Half Log.

| Height Above GroundFT) | Diameter Inside Bark rounded to nearest inch | Cubic Foot Volume | Board Scribn | Foot Volume <br> Int'nl 1/4 |
| :---: | :---: | :---: | :---: | :---: |
| $87.0 \longrightarrow$ - 6 " | dib |  |  |  |
| 83.5 | 7.0 |  |  |  |
|  |  | 7.4 | 3.0 | 3.0 |
| 67.0 | 11.0 |  |  |  |
|  |  | 15.1 | 7.0 | 8.0 |
| $50.5 \gg$ | 15.0 |  |  |  |
|  |  | 24.0 | 14.0 | 16.0 |
| $34.0 \rightarrow$ 元 | 18.0 |  |  |  |
|  |  | 31.6 | 21.0 | 23.0 |
| $17.5>$ | 20.0 |  |  |  |
|  |  | 40.5 | 28.0 | 29.0 |
| $\xrightarrow{4.5} \longrightarrow$ | 23.0 |  |  |  |
| Ground FT. $\qquad$ level | Total Volu $.5 \mathrm{ft})$ | me $\overline{18.6}$ | 73.0 | 79.0 |

51.37 - Exhibit 07

Summarization of Volume by Segmentation Rule

51.4 - Automated Procedures. To minimize computational errors, use a computer program to calculate the segments, cubic, and board foot volumes. Ensure that the program is specific for each Region and Forest, and has embedded in it the standards approved by the Regional Forester (sec. 04.2).
51.5 - Metric. Metric units have the same characteristics as cubic feet. Use cubic meters for volume. Use formulas that have the same characteristics as the cubic foot formulas, except apply the constant K :
$K=.00007854$
52 - VOLUME ESTIMATOR USES AND CAPABILITIES.
52.1 - Types of Volume Estimators.
52.11 - Profile Equations. Use profile equations that describe the stem form whenever possible. Profile equations are superior to other tree volume estimators because they calculate diameter all along the stem. Use profile equations not only in determining tree volume, but also in determining log volume, grade, and value. Use them to determine changes due to bucking practices and different top diameter specifications. For a technical description of profile equations see chapter 30.
52.12 - Direct Volume Estimators. These may include volume tables, $\mathrm{D}^{2} \mathrm{H}$ equations, alignment charts, logarithmic equations, local volume equations, or tarif tables. For a technical description of direct volume estimators see chapter 30. Avoid new development of this type of estimator.
52.13 - Aerial Estimators. Apply aerial photo volume estimation through a double sampling technique. Volume estimators for use with photo measurements often have parameters of total tree height, crown diameter, and crown area. Conduct field sampling of ground plots and compare with the photo plots of the same area. Consider bias involved in photo volume estimators to be resolved by double sampling.
52.14 - Biomass Estimators. Biomass estimators may be used to find the amount of wood, bark, and foliage in standing material, regardless of quality or size. Biomass is measured in weight; however, use the traditional predictors, diameter and height, and regression analysis to predict weight. Use the diameter at the base of the live crown as the best indicator of crown weight.
52.2 - Flexibility. The flexibility of a volume estimator depends upon the type of estimator, its complexity, available computer capacity, and its intended use. Flexibility refers to range and ease of application. See chapter 30 for discussion of flexibility of specific volume estimators.
52.3 - Limitations of Estimators. Identify the estimator's limitations which may be geographical, technical, data related, or practical. Take reasonable care so that resulting volume estimates are not erroneous. See section 33.32 Exhibit 01 for a ranking of various volume estimator models.

TIMBER VOLUME ESTIMATOR HANDBOOK

53 - APPLYING THE ESTIMATORS.
53.1 - Typical Trees. A typical tree is normally formed and is described by its height and diameter breast height. Most stem profile equations are developed for normally formed trees. Apply them directly to normal trees.
53.2 - Nontypical Stands or Trees. To apply a volume estimator to a nontypical (abnormal) tree or stand, determine which stand or tree conditions were excluded from the development of the normal volume estimators. For example, forked or deformed trees or defective stands or local stand variations may have been excluded from volume estimator development. Use felled tree data, a dendrometer or laser device to develop a location or stand correction ratio as necessary to adjust stand volumes for abnormal trees or stands. Use localized procedures such as "fall, buck, and scale" or "correction factor scaling."
53.21 - Defective Stands. Identify highly defective stands which may vary in form from normal stands. For example, defect may cause excessive butt swell. Develop appropriate adjustments to the volume estimates locally such as developing a local regression equation based on normally formed trees in the stand
53.22 - Locality Based Form Variations. Identify local conditions such as elevation, site quality, age class, or disease that have major influence on tree form. If consistent, important variation occurs use calibration procedures (sec. 43). Develop a new estimator (sec. 32) if justified by an extensive condition and a large amount of cruising or inventory to be done. If differences are minor or on an individual stand or tree basis, use procedures in sections 53.2-53.22.
53.3 - Tree Height. For general purposes, use total height in feet as the basis for applying volume estimators.
53.31 - Missing and Abnormal Tops. Determine heights for use of stem profile equations when trees are forked, or have missing or deformed tops. Estimate what the normal tree height of the tree would have been without the abnormality, and apply the estimator directly. Do this by developing a local regression equation based on normally formed trees in the stand, or by the cruiser making an estimate of total height. Estimate the missing or reduced volume as defect (sec. 53.5).
53.32 - Height to a Merchantable Top. In the case of hardwoods, height may have been measured to a merchantable top. If a tree species typically has no well defined bole above this point, do not measure total height. Do not measure total height when it is impossible or highly inaccurate. Use stem profile equations constructed from the ground to a merchantable height. Apply the equations by knowing the merchantable diameter and estimating height to the given diameter, or by measuring the diameter at the merchantable top using a dendrometer or laser device when merchantable diameter typically occurs where the bole ends at the base of the crown.
53.33 - Tree Height Measured in Logs. Avoid estimating heights in units of length such as 16 -foot logs. Use shorter units such as feet
or meters. Recognize, however, that some estimates of tree height are in logs.

Stem profile equations utilize total height in feet, and if log height is collected, transform it to total height in feet for use in a stem profile equation. Develop a local regression of log height verses total height and base this regression on trees which were actually measured for total height.
53.4 - Utilization Specifications. Adjust volume estimates and estimators when changes in utilization specifications occur. These may affect merchantable log lengths, average tree volume, number of logs per unit of measure, and volume per acre. Even though volume tables based on a single fixed top diameter or set log lengths cannot be adjusted easily or accurately for changes in utilization specifications, the estimators must be adapted to the specifications in use. See field unit supplements to the Timber Sale Preparation Handbook, FSH 2409.18 , section 54.5 or other local direction for applicable utilization specifications.
53.5 - Defect and Net Volume. Almost all volume estimators are constructed to determine gross volume. Therefore, determine defect and the net volume, as needed for timber sales or inventory purposes according to the directions in the Cubic Scaling Handbook, FSH 2409.11a, the National Forest Log Scaling Handbook, FSH 2409.11, and the Timber Cruising Handbook, FSH 2409.12.
53.6 - Appraisal Data Needs. Recognize that different geographic areas may require variations of a volume estimator based on appraisal practices. Use estimators to predict overrun, tree quality or grade, and lumber tally volumes, as needed in timber appraisals. Adapt the volume estimators accordingly. Table 01 lists some of the common appraisal data elements that may be computed or measured in volume estimation processes. See FSH 2409.18, sections 45-49.4 for appraisal direction.
53.6 - Table 01

Common Appraisal Items That Can Be Computed in Volume Estimation Processes.

1. Average DBH.
2. Average tree height.
3. Average merchantable tree height.
4. Volume per tree.
5. Woods and scaling defect.
6. Logs per thousand Board Feet (MBF) (16 or 32 foot).
7. Total number of logs (16 or 32 foot).
8. Overrun
9. Tree quality.
10. Largest log.
11. Largest tree.
12. Log grade.
13. Height to live limb.
14. Average volume per acre.
15. Total number of trees.

TIMBER VOLUME ESTIMATOR HANDBOOK

## Contents

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CHAPTER 60 - FITTING

61 - FITTING PROCESS. Use fitting to adapt the volume estimator to subsets of data (sec. 35.4).
61.1 - General Description - Least Squares Regression. Use the least squares regression procedure in fitting a stem profile equation to a set of stem measurements. Estimate the coefficients of the stem profile equation in the fitting process. Use the least squares approach to minimize the sum of squared differences between the actual value of the dependent variable and the value predicted by the equation. The minimized value is called the sum of squared error (SSE).

$$
S S E=\sum^{n}\left(y^{-} \hat{y}\right)^{2}
$$

where:

$$
\begin{aligned}
& y=\text { observed value of the dependent variable } y \\
& \hat{y}=\text { value of } y \text { predicted by equation }
\end{aligned}
$$

When a stem profile model is linear in the coefficients, use ordinary least squares (OLS) regression fitting procedure. When a model is nonlinear in some or all of the coefficients, use nonlinear least squares (NLLS) regression. Regard the minimum SSE obtained by fitting a particular model to data as the minimum for that model only. Consider fitting another, more flexible model which may result in a smaller SSE. Do not regard SSE or residual mean square (RMS) as the only goodness of fit criterion. Consult regression textbooks that describe procedures for analyzing residuals to detect any of several conditions which are not characteristic of well-fitted regression.
61.11 - Ordinary Least Squares (OLS) Regression. Use ordinary least squares (OLS) regression with simple or complex polynomial stem profile models which are linear in the coefficients. An example of a simple linear stem profile model is that of Kozak and others (sec. $08 ; ~ s e c .33 .32)$. This model is linear in the coefficients $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$. A more complex linear model is that of Bruce and others (sec. 08; sec. 33.32). This more complex model is linear in all of its coefficients, $b_{1}$ to $b_{6}$. Use ordinary least squares for nonlinear models if they can be transformed into a linear form. For example, the simple nonlinear model of Ormerod, (sec. 33.32) which is nonlinear in the coefficient $b_{1}$ may be converted to a linear form using a logarithmic transformation.

All major statistical software packages contain OLS regression procedures. Software packages and their respective OLS procedures are listed in Table 01. Refer to the respective user's manual for the details in using these procedures.

61.11 - Table 01<br>Major OLS Statistical Software

Statistical Package
BMDP (1981)
SPSS (1986)
SAS (1985
IMSL (1982)
SYSTAT (1987)
61.12 - Nonlinear Least Squares (NLLS) Regression. For most stem profile equations, use NLLS regression procedures. Many of the more complex models, including segmented models, are nonlinear relative to some or all of the parameters. Avoid complexity whenever possible, but recognize that a tree stem is a complex solid object, and that simpler stem profile models do not describe the tree as well as flexible and complex models do. Recogize that the NLLS procedure is more complex than OLS. Satisfy NLLS requirements, as necessary, such as providing starting values for the coefficients, placing bounds on the coefficient values, and possibly providing first order partial derivatives with respect to each of the coefficients.

Acquire working knowledge of the model parameters before attempting to fit final coefficients. Recognize that due to starting values and the nature of the SSE surface, the NLLS algorithm may converge to a local minimum instead of the desired global minimum of SSE. Be aware that there is no guarantee that the iterative NLLS procedure will converge to the global minimum of SSE; therefore, use one of several methods to establish starting coefficient values for the stem profile model to be fit. For an existing model, use coefficient values found by others for the same model as reasonable starting values, even though they may be for a different species or geographic area. In the absence of other results, use SSE for different values of the coefficients. Systematically vary the coefficients to provide a range of SSE's. Use the statistical software package (SAS), or an equivalent, to perform the systematic varying of coefficients. Plan to calculate SSE for many combinations of coefficients and for the expense of doing so. Use the coefficient values associated with the lowest $S S E$ as starting values in the NLLS regression procedure.

Provide upper and lower bounds for the coefficients to make the NLLS procedure converge in fewer iterations. Use caution in setting bounds on the coefficients since a coefficient estimated in NLLS as one of its pre-set bounds may be an indicator of a restrictive bound limiting the procedure from finding the global minimum.

Provide first-order partial derivatives with respect to each coefficient in the stem profile model depending on the specific procedure used. Employ a procedure using analytical derivatives if one is available since derivative-free algorithms are usually less efficient than algorithms requiring derivatives. However, derivative-free algorithms may be used for models with difficult-to-derive partial derivatives, since errors in deriving the first order partial derivatives impact the NLLS fitting procedure.

Examples of the first order partial derivatives for two common stem profile models are given in tables 01 and 02.

$$
61.12 \text { - Table } 01
$$

Partial Derivative, General Form of the Ormerod Stem Profile Model

$$
\frac{d^{2}}{D^{2}}=b_{1}\left(\frac{h}{H}-1\right)+b_{2}\left(\frac{h^{2}}{H^{2}}-1\right)+b_{3}\left(a_{1}-\frac{h}{H}\right)^{2} I_{1}+b_{4}\left(a_{2}-\frac{h}{H}\right)^{2} I_{2}
$$

where:

$$
I_{i}=1 \text { if } h / H \leq a_{i}
$$

$$
=0 \text { if } h / H>a_{i}, \text { for } i=1,2
$$

if $y=\frac{d^{2}}{D^{2}}$ then the partial derivitives with respect
to the six coefficients $b_{1}, b_{2}, b_{3}, b_{4}, a_{1}$ and $a_{2}$ are

$$
\begin{array}{rlrl}
\frac{\partial y}{\partial b_{1}} & =\frac{h}{H}-1 & \frac{\partial y}{\partial b_{4}} & =\left(a_{2}-\frac{h}{H}\right)^{2} I_{2} \\
\frac{\partial y}{\partial b_{2}} & =\frac{h^{2}}{H^{2}}-1 & \frac{\partial y}{\partial a_{1}} & =2 b_{3} I_{1}\left(a_{1}-\frac{h}{H}\right) \\
\frac{\partial y}{\partial b_{3}}=\left(a_{1}-\frac{h}{H}\right)^{2} I_{1} & \frac{\partial y}{\partial a_{2}} & =2 b_{4} I_{2}\left(a_{2}-\frac{h}{H}\right)
\end{array}
$$

### 61.12 - Table 02

Partial Derivative, Segmented Model of Max and Burkhart

$$
\begin{gathered}
\frac{d^{2}}{D^{2}}=b_{1}^{2}\left(\frac{H-h}{H-4.5}\right)^{2 b_{2}} \\
\text { if } y=\frac{d^{2}}{D^{2}} \text {, the partial derivitives } \\
\text { with respect to } b_{1} \text { and } b_{2} \text { are } \\
\frac{\partial y}{\partial b_{1}}=2 b_{1}\left(\frac{H-h}{H-4.5}\right)^{2 b_{2}} \\
\frac{\partial y}{\partial b_{2}}=2 b_{1}^{2}\left(\frac{H-h}{H-4.5}\right)^{2 b_{2}} \ln \left(\frac{H-h}{H-4.5}\right)
\end{gathered}
$$

61.2 - Statistical Procedures. Table 01 lists the NLLS regression procedures available in each of the major statistical packages.
61.2 - Table 01

Major NLLS Statistical Software

## Statistical Package

BMDP (1981)

SPSS (1986)
in
SAS (1985)

IMSL (1982)
SYSTAT (1987)

NLLS Procedures
P3R (derivatives required)
PAR (derivative-free)
None available at the present, possibly available future versions.

NLIN (derivatives required or derivative-free)

ZXMIN (derivative-free)
NONLIN (derivative free)

Refer to the respective user's manual for the details in using these procedures.

62 - VARIABLES OTHER THAN $D, H$, and $h$ IN STEM PROFILE EQUATIONS.
Account for most of the variation in the stem profile using $D, H$, and h. However, incorporate other variables into a stem profile model when effective in increasing the accuracy and precision of predictions made for the model. Consider other variables often used such as measures of stem form, crown measurements, and indicators of site quality and geographic location.

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62.1 - Crown and Form Measurements. Because of the importance of the live crown on stem taper, consider crown measurements (especially crown ratio) as an additional variable in stem profile models. For more detailed discussion of using crown measurements in a stem profile model refer to Dell, Burkhart and Walton, Newberry and Burkhart, and Valenti and Cao (sec. 08). Trees with the same D and H may have different forms but most stem profile equations fit an average profile for trees with the same dimensions. Using a form measurement as a variable may improve the fit of the model. Form quotients (that is a ratio of a fixed upper stem diameter to Dbh, such as Girard form class) or $H / D$ ratios are commonly used form measurements. Refer to Matney and Sullivan, Schlaegel, and Czaplewski and McClure (sec. 08) for previous work done in this area. Improvement in predictive value of a model requiring an upper stem measurement which may be substantial is gained at the cost of measuring that upper stem diameter on trees with which it will be used. Using the H/D ratio as a proxy for form quotient avoids the need for an additional measurement.
62.2 - Geographic Location and Site Quality Indicators. Use geographic location and indicators of site quality when they may be able to explain some additional variation in stem profile. Geographic variation in the relationship between stem profile and volume has been shown to exist; therefore, expect that geographic location may account for some variation. Likewise, site variables, such as ecological habitat, elevation, slope, aspects, and the like because their relationship to stem growth might be useful for incorporation into a model. Refer to Larson for a brief outline of the work that has been completed (sec. 08). Explore these variables, if possible, as additional predictor variables in stem profile models.
62.3 - Additional Variables. Review the several methods of incorporating additional predictor variables into stem profile models. Select a method that preserves the integrability of the equation for efficient volume determination.
62.31 - Form Quotient. To incorporate form quotient into a stem profile model, condition the model to exactly predict the upper stem diameter corresponding to the form quotient. Condition it by mathematically manipulating the equation so that it meets the desired criteria and integrate the model for volume determination if possible.
62.32 - Fitting Classes of Data. Consider incorporating additional variables into a model by dividing the data set into classes based on the variable of interest and then fitting the model to each class of data. Use this approach as an exploratory method in determining possible relationships between variables and stem profile coefficients, then use the two-stage final fitting procedure described in section 62.33. If many classes exist, maintaining the right regression coefficients for each class will be more burdensome and deciding how to divide the data into valid classes without knowing the underlying relationships is difficult. If done incorrectly, there may be too few classes to adequately describe relationships, or there may be too many classes resulting in an unnecessary workload in the fitting process.
62.33 - Two-Stage Fitting. Use a two-stage fitting approach in most situations requiring incorporation of additional predictor variables.

Fit the stem profile equation to each tree in the data set or to small groups of trees with the same dimensions and characteristics. Relate the coefficients obtained in the first stage to the additional variables of interest by using least squares regression techniques such as stepwise or all possible subsets regression procedures in the second stage. After the second stage model forms are determined, fit the stem profile model, with the second stage equations included, to the full data set for final coefficient determination. Use this procedure to maintain the integrability of the stem profile model and to adequately describe the relationships between additional variables and stem profile.


[^0]:    * Too few observations to compute this statistic.

