

**Crown ratio modeling using stand density index  
and the  
Weibull distribution**

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## Introduction

Prognosis (Stage 1973) is an individual tree distance independent growth and yield model developed for planning management regimes in Northern Rocky Mountain forests. Geographic variants of Prognosis have been calibrated for many areas of the Western United States. The WO Timber Management Staff in Ft. Collins, Colorado, in conjunction with Forest Service Regions 5 and 6, are currently developing a Prognosis variant for South Central Oregon / Northeast California (SORNEC). As part of this effort, a better way to estimate crown ratio, and crown ratio change in response to changing stand conditions, is being examined.

The latest release of Prognosis (Wykoff 1985; also see Wykoff, Crookston, and Stage 1982) predicts crown ratio from habitat type, basal area, crown competition factor, dbh, tree height, and percentile in the basal area distribution. The equation is well behaved but seems rather insensitive to changing stand density. Stand Density Index, or SDI, (Reineke 1933) was used to solve a similar problem with estimating mortality in the SORNEC variant. The resulting mortality function is very sensitive to changing stand density conditions. The SDI approach is also being taken in estimating crown ratio in an effort to get a similar sensitivity.

## Estimating Stand Density Index

Eleven species are recognized in the SORNEC variant: western white pine, sugar pine, incense cedar, Engelmann spruce, white fir, Shasta red fir, lodgepole pine, juniper, mountain hemlock, Douglas-fir, and ponderosa pine. Existing data from a variety of sources were used to calibrate SORNEC, however, only data for juniper, Douglas-fir, mountain hemlock, lodgepole pine, Shasta red fir, and white fir were available to estimate SDI. Plots were constructed for each species showing the relationship between quadratic mean stand diameter and stand density (trees/acre). Plots for white fir and Douglas-fir are shown in Figures 1a and 1b, respectively. These species were chosen to illustrate the difference between a species with a relatively constant crown ratio, Douglas-fir, versus a species with a highly variable crown ratio, white fir. The same relationships on a logarithmic scale are shown in Figures 2a and 2b.

Equation (1) shows the relationship between number of trees/acre, (N), and quadratic mean stand diameter, (QMD), used by Reineke. Reineke found the

$$(1) N = a QMD^b$$

"b" coefficient to be relatively constant at -1.605 for most species. Using this approximation for "b" and transforming to logarithmic scale gives equation (2).

$$(2) \ln(N) = \ln(a) - 1.605 \ln(QMD)$$

A line with -1.605 slope was drawn on the logarithmic scale plots to form an upper bound, or maximum, SDI. These are shown on Figures 2a and 2b. It is interesting to note how well the -1.605 slope matches the data. The corresponding line in real scale is shown on Figures 1a and 1b.

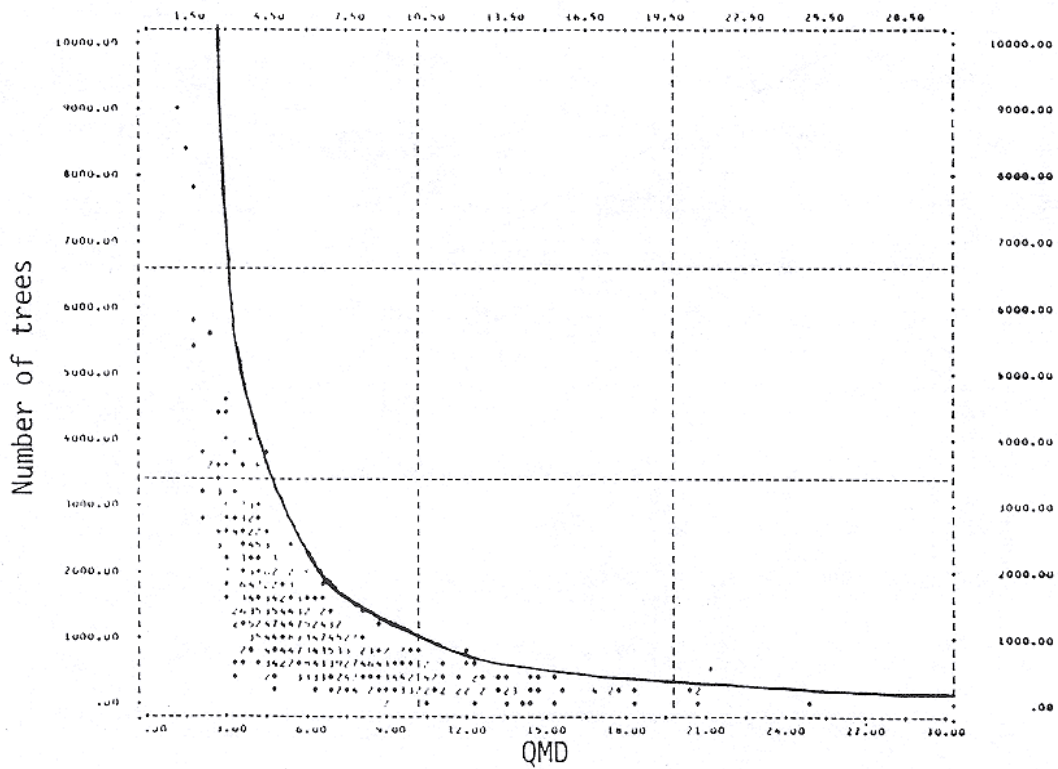


Figure 1a. Stand density versus quadratic mean stand diameter for white fir in Southcentral Oregon and Northeast California.

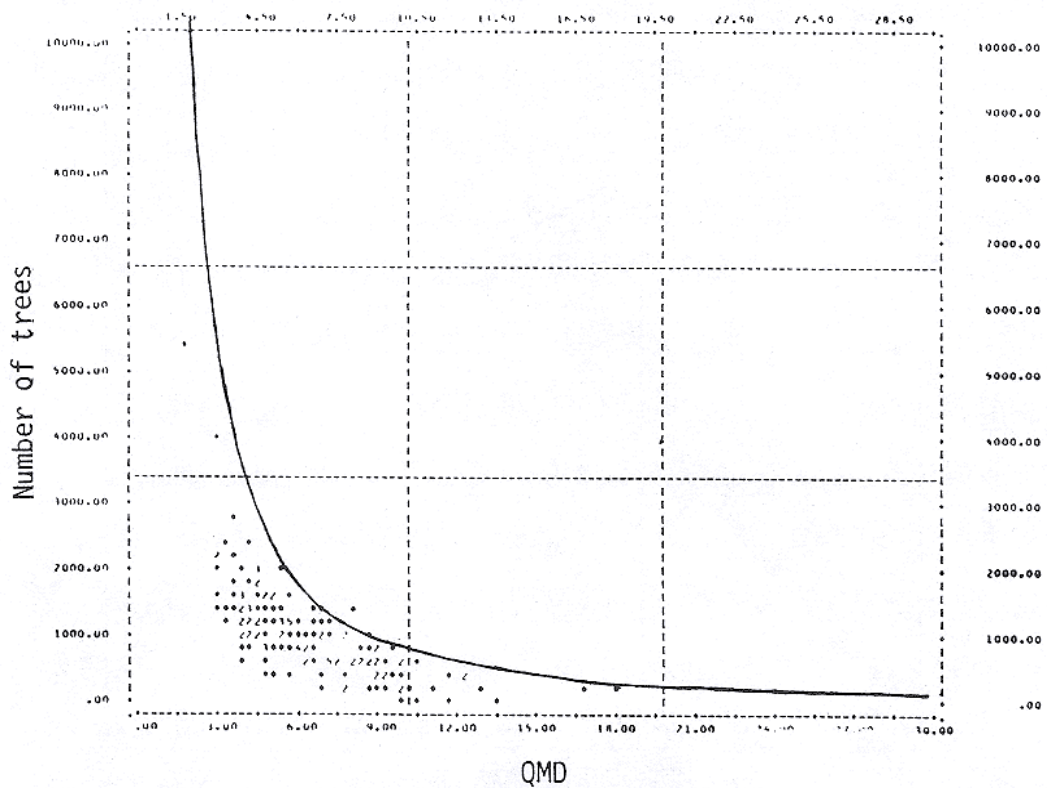


Figure 1b. Stand density versus quadratic mean stand diameter for Douglas fir in Southcentral Oregon and Northeast California.

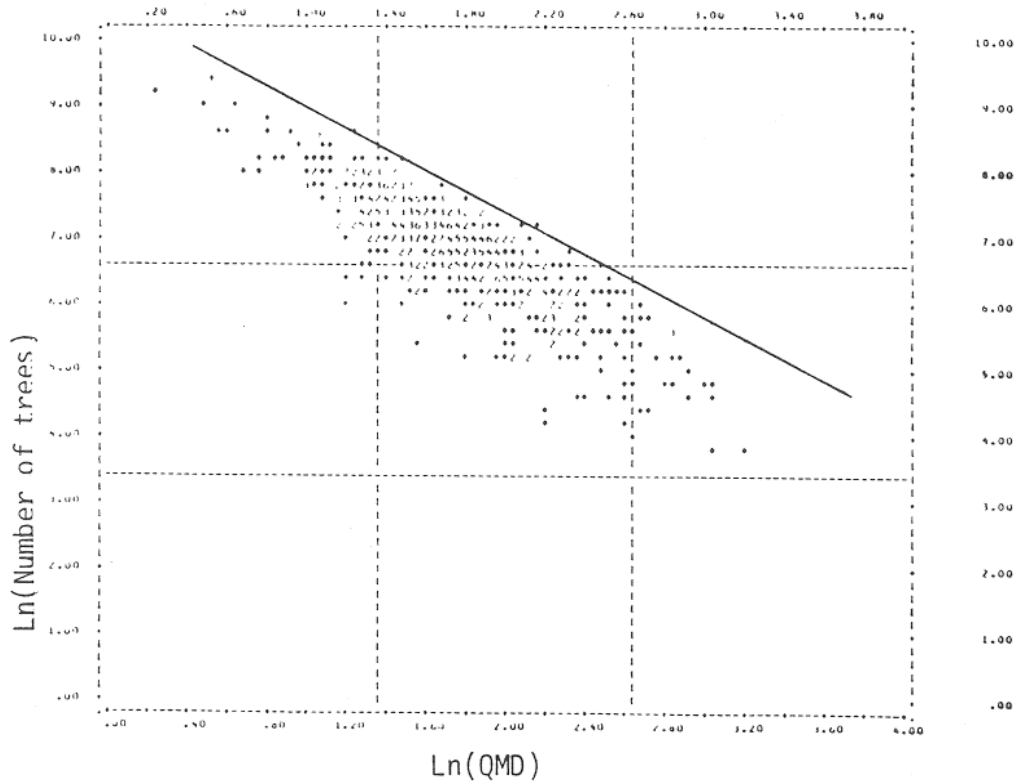


Figure 2a. Stand density versus quadratic mean stand diameter in logarithmic scale for white fir in Southcentral Oregon and Northeast California.

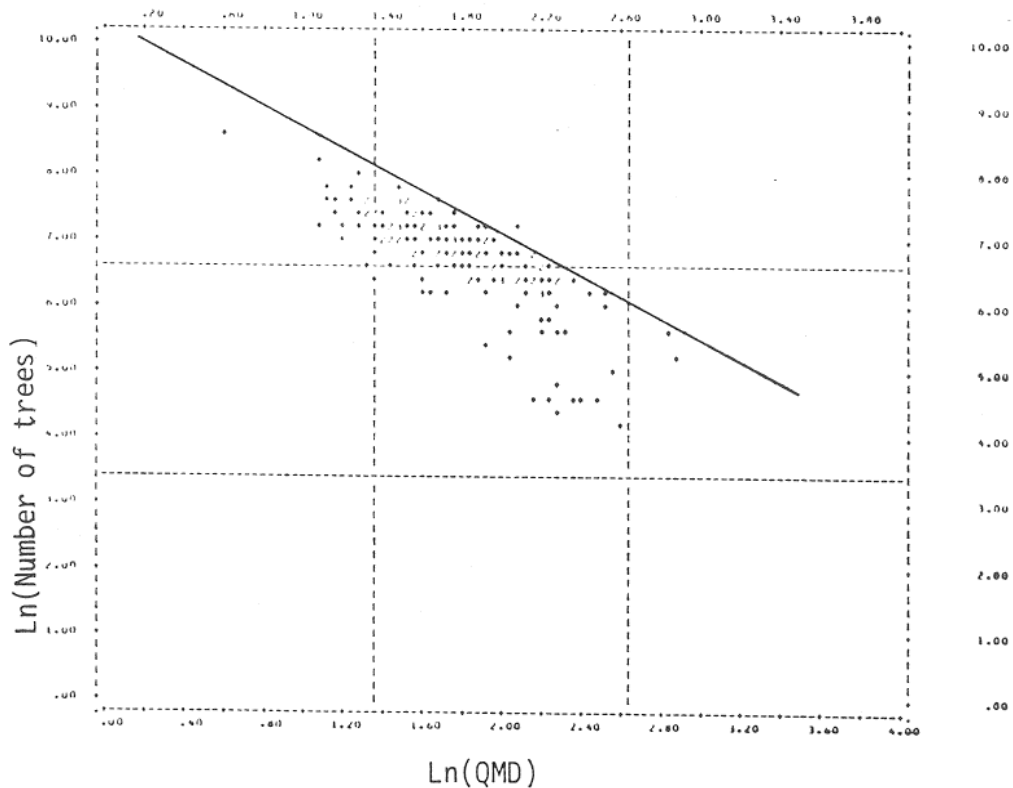


Figure 2b. Stand density versus quadratic mean stand diameter in logarithmic scale for Douglas fir in Southcentral Oregon and Northeast California.

To estimate the "a" coefficient, a point was picked on either the real scale line or the logarithmic scale line and substituted into equation (1) or (2) as appropriate. For white fir the point (2-7183,6.25) was used with equation (2), and for Douglas-fir (6.00,1710) was used with equation (1). The maximum SDI value was, then calculated by solving the resulting equation for QMD = 10.0 . Equation coefficients and corresponding SDI values, by species, are shown in Table 1. In theory, stand density peaks at about 85 percent of maximum (Smith 1984, Long 1984, Johnson 1984). These 85 percent values are also shown in Table 1.

Species	"a" (equation 1)	"a" (equation 2)	Max. SDI	85% level
Juniper	24834.8	10.12000	617	524
Douglas-fir	30333.9	10.32002	753	640
Mountain hemlock	30511.3	10.32585	758	644
Lodgepole pine	35478.3	10.47668	881	749
Shasta red fir	40654.8	10.61287	1009	858
White fir	40654.8	10.61287	1009	858

Table 1. Coefficients and corresponding SDI values, by species, for South Central Oregon / Northeast California.

#### Estimating Weibull parameters

The Weibull distribution is described by the probability density function shown in equation (3) (Johnson and Kotz 1970). Parameters were estimated by

$$(3) f(x) = \frac{c}{b} \left( \frac{x-a}{b} \right)^{(c-1)} e^{-\left(\frac{x-a}{b}\right)^c}$$

first constructing a table of tree frequencies by crown ratio code (1=0-10%, 2=11-20%, etc.) and relative SDI (stand SDI / 85% level) for each species. Weibull parameters were then estimated for each relative SDI group with more than 10 observations. Frequency tables and estimated parameters for white fir and Douglas-fir are shown in Tables 2a and 2b. Note the decreasing trend in mean crown ratio as relative SDI increases for white fir, while the trend is less pronounced for Douglas-fir.

Data were available for 9 of the 11 species for this portion of the study. Only Engelmann spruce and Juniper lacked enough observations to estimate the Weibull parameters. Since maximum SDI values were not available for all 9 of these species, the SDI value for lodgepole pine was used for

Rel. SDI	Crown ratio code									Total	Mean Crown Ratio	Weibull Parameters		
	1	2	3	4	5	6	7	8	9			a	b	c
0- 10	0	1	2	2	0	4	2	6	4	21	6.57	0.0	7.30	3.70
11- 20	1	6	12	16	16	29	54	38	22	194	6.43	0.0	7.09	4.17
21- 30	1	15	30	43	61	79	89	65	44	427	6.12	0.0	6.78	3.78
31- 40	1	12	48	119	176	204	181	101	46	888	5.90	0.0	6.50	4.16
41- 50	4	33	78	151	216	224	178	88	34	1006	5.56	0.0	6.15	3.76
51- 60	7	48	84	201	252	217	215	81	14	1119	5.36	0.0	5.94	3.74
61- 70	2	31	118	174	160	132	92	51	6	766	4.98	0.0	5.55	3.38
71- 80	0	41	91	166	157	123	62	27	7	674	4.83	0.0	5.38	3.42
81- 90	1	14	28	93	71	43	21	9	0	280	4.70	0.0	5.21	3.71
91-100	1	9	19	39	36	15	5	2	0	126	4.39	0.0	4.86	3.64
101-110	2	7	23	43	27	14	6	2	0	124	4.31	0.0	4.78	3.44
111-120	0	8	7	12	14	15	3	0	0	59	4.51	0.0	5.02	3.60

Table 2a. Number of trees by crown ratio code and relative SDI for white fir, and the corresponding Weibull parameters.

Rel. SDI	Crown ratio code									Total	Mean Crown Ratio	Weibull Parameters		
	1	2	3	4	5	6	7	8	9			a	b	c
0- 10	0	0	0	0	0	0	0	0	0	0	0.00			
11- 20	1	0	2	11	7	7	3	3	0	34	5.09	1.0	4.61	3.04
21- 30	0	0	2	3	5	4	5	5	2	26	6.15	1.0	5.75	3.41
31- 40	1	0	1	6	5	3	3	6	1	26	5.73	1.0	5.36	2.86
41- 50	0	0	5	8	22	15	12	16	8	86	6.17	1.0	5.77	3.45
51- 60	0	1	3	10	26	29	18	9	5	101	5.92	1.0	5.44	3.77
61- 70	2	3	13	17	13	21	15	11	5	100	5.44	1.0	5.03	2.59
71- 80	0	2	9	18	14	16	19	1	1	80	5.24	1.0	4.74	3.12
81- 90	0	0	1	12	17	18	4	5	0	57	5.47	1.0	4.93	3.99
91-100	0	2	4	14	15	31	20	4	0	90	5.61	1.0	5.09	4.05
101-110	0	11	6	21	12	17	9	5	2	83	4.90	1.0	4.41	2.30
111-120	0	0	2	8	20	1	1	0	0	32	4.72	1.0	4.02	5.09

Table 2b. Number of trees by crown ratio code and relative SDI for Douglas-fir, and the corresponding Weibull parameters.

white pine and ponderosa pine, and the white fir value for sugar pine and incense cedar. The *I*lan (location) coefficient was found to be nearly constant across relative SDI groups for a given species. Consequently, it was set to a constant and the remaining parameters recomputed. The "b" (scale) parameter showed a high correlation with the mean crown ratio, while the "c" (shape) coefficient showed no consistent relationship to mean crown ratio. These parameters are shown in Tables 2a and 2b, and graphically in a later section.

#### **Relating mean stand crown ratio to SDI**

Using mean stand crown ratio (MCR) values and midpoints of the relative SDI classes (RSDI) from frequency tables such as those shown in Tables 2a and 2b, least squares regression equations were developed to predict MCR from RSDI. The simple linear model  $MCR = d_0 + d_1 * RSDI$  was used, and data from all eleven species were available for this regression. These relationships are shown graphically in Figures 3a and 3b for white fir and Douglas-fir, and the regression results for all species are listed in Table 3. Note that the crown ratio estimators for Douglas-fir, Engelmann spruce, and juniper are constants.

Species	d0	d1	r	s.e.
White fir	6.613	-0.022	.958	.172
Lodgepole pine	5.507	-0.018	.798	.363
Ponderosa pine	6.412	-0.020	.760	.483
Incense cedar	6.174	-0.018	.862	.230
Douglas-fir	5.527	0.000		
Mountain hemlock	7.451	-0.024	.682	.481
Shasta red fir	6.128	-0.013	.414	.485
White pine	7.168	-0.024	.648	.463
Sugar pine	6.597	-0.020	.778	.247
Engelmann spruce	6.774	0.000		
Juniper	7.238	0.000		

Table 3. Regression results for the relationship  $MCR = d_0 + d_1 * SDI$ , by species, for South Central Oregon and Northeast California.

#### **Relating the Weibull parameters to crown ratio**

As stated previously, the Weibull "a" parameter was considered constant for a given species. Equations predicting the Weibull "b" parameter from mean crown ratio were developed through ordinary least squares regression. Plots of this relationship for white fir and Douglas-fir, and the resulting regression equations are shown in Figures 4a and 4b. Since no clear pattern

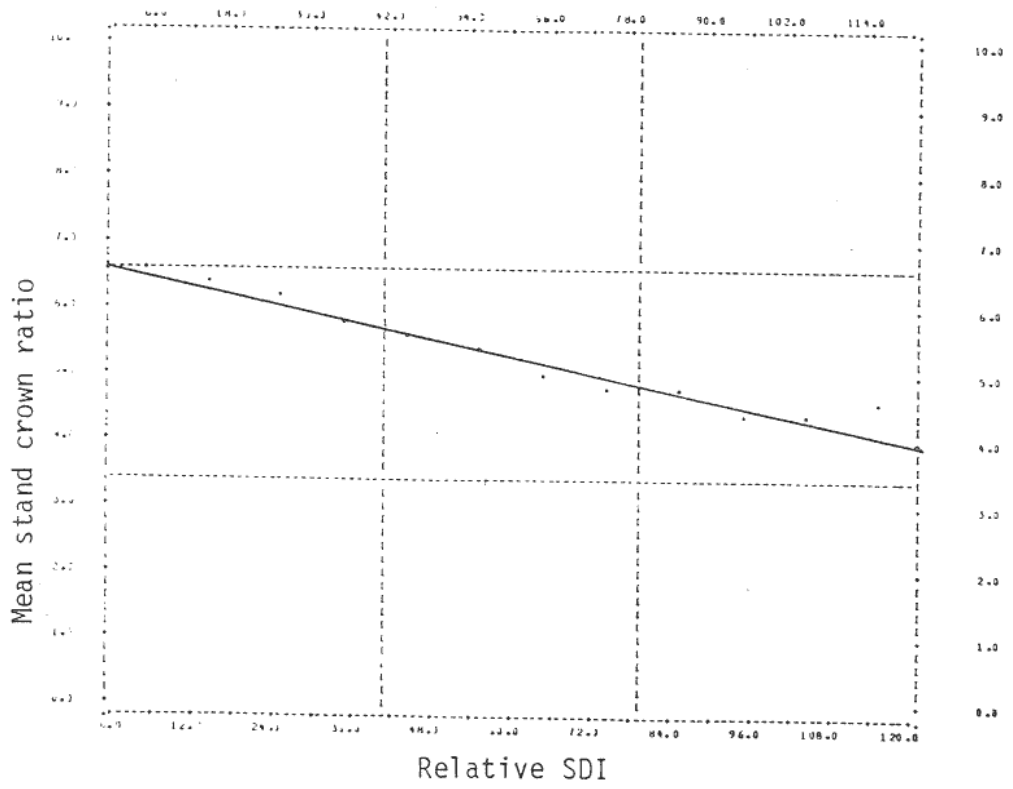


Figure 3a. Mean stand crown ratio versus relative SDI for white fir in Southcentral Oregon and Northeast California.

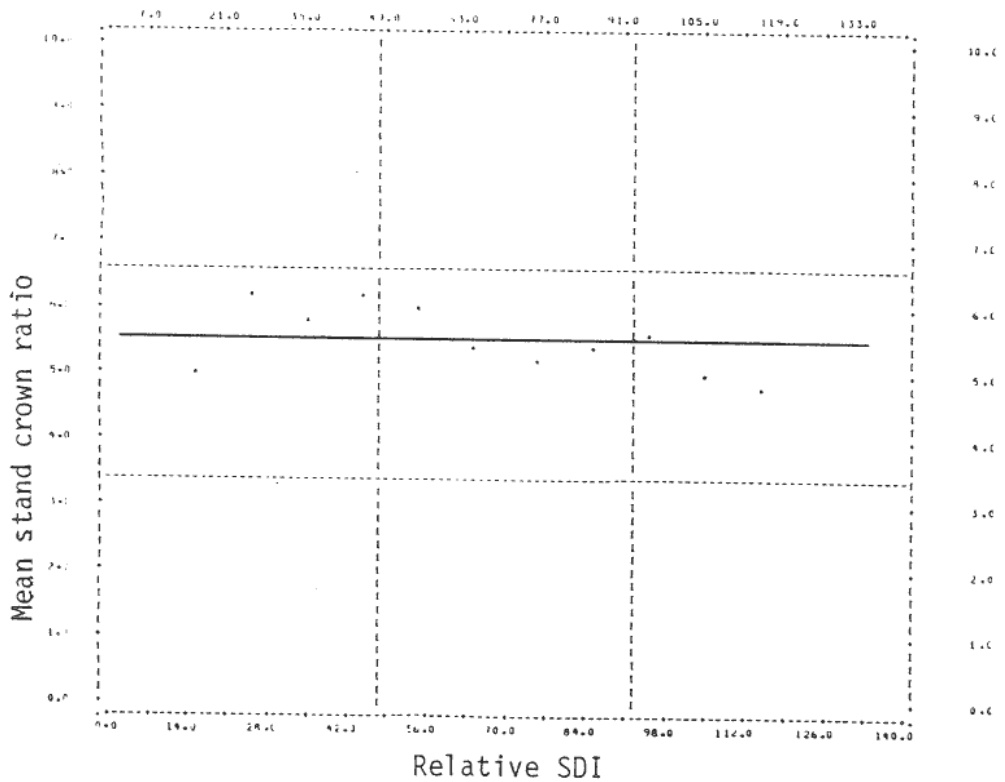


Figure 3b. Mean stand crown ratio versus relative SDI for Douglas fir in Southcentral Oregon and Northeast California.



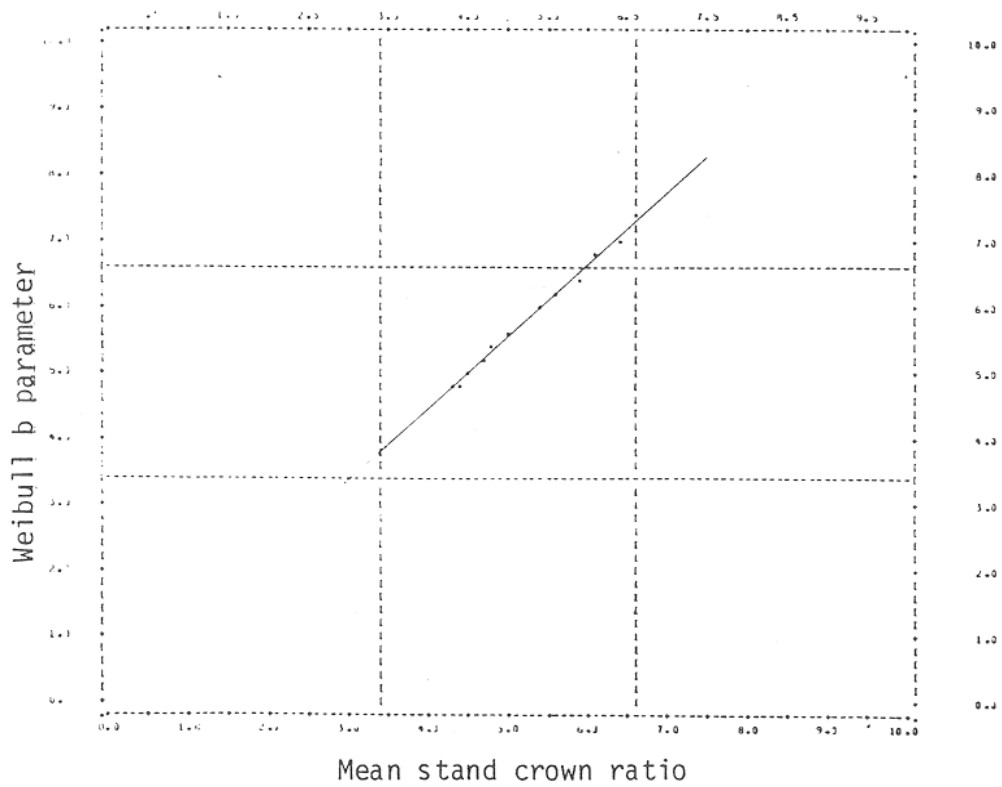


Figure 4a. Weibull scale parameter versus mean stand crown ratio for white fir in Southcentral Oregon and Northeast California.

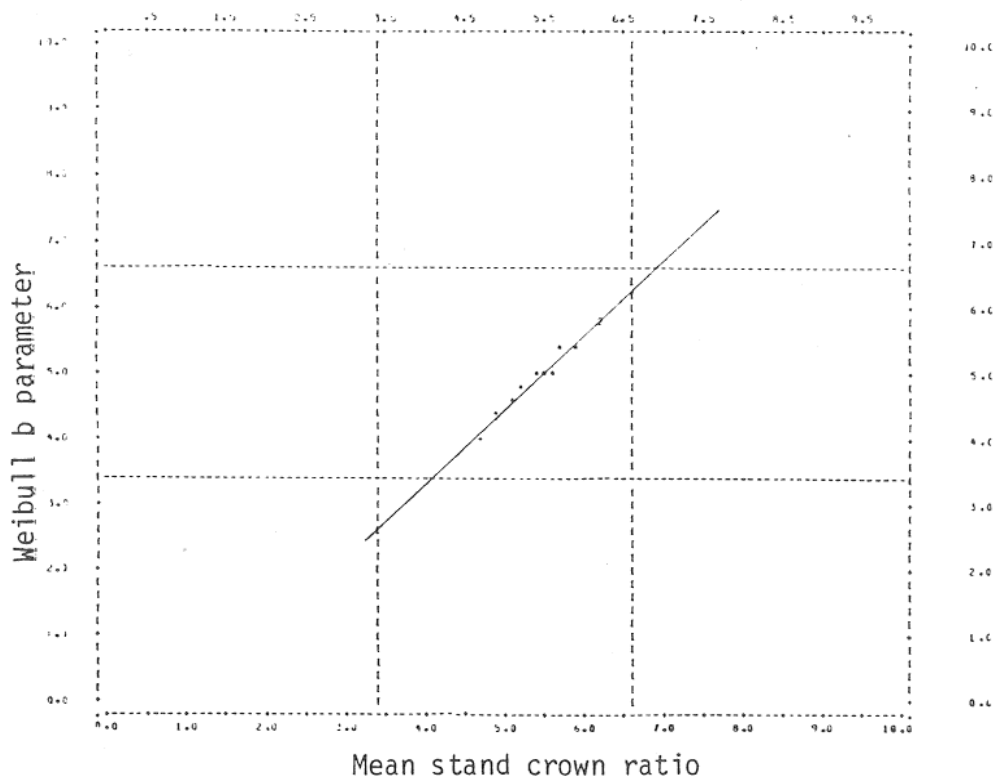


Figure 4b. Weibull scale parameter versus mean stand crown ratio for Douglas fir in Southcentral Oregon and Northeast California

emerged for the Weibull "c" parameter, it too was considered a constant, and estimated as the arithmetic average of "c" values across relative SDI groups for a species. The relationship of "c" with mean crown ratio is shown in Figures 5a and 5b for white fir and Douglas-fir respectively. The estimated parameters for the various species in this study are shown in Table 4. In the SORNEC variant, parameters for Shasta red fir were also used for Engelmann spruce, while those for lodgepole pine were used for juniper.

Species	Weibull Parameters			"b" equation	
	a	b	c	r	s.e.
White fir	0.0	0.066 + 1.096 * CR	3.71	.999	.022
Lodgepole pine	0.0	0.076 + 1.102 * CR	3.01	.999	.032
Ponderosa pine	0.0	0.249 + 1.048 * CR	4.36	.999	.030
Incense cedar	1.0	-1.386 + 1.168 * CR	3.02	.991	.068
Douglas-fir	1.0	-1.193 + 1.129 * CR	3.42	.986	.068
Mountain hemlock	1.0	-0.941 + 1.083 * CR	3.47	.995	.063
Shasta red fir	1.0	-0.916 + 1.065 * CR	3.50	.976	.105
White pine	2.0	-2.127 + 1.105 * CR	2.77	.995	.060
Sugar pine	2.0	-2.273 + 1.124 * CR	3.34	.997	.034

Table 4. Weibull parameters for modeling crown ratio in the SORNEC variant of Prognosis.

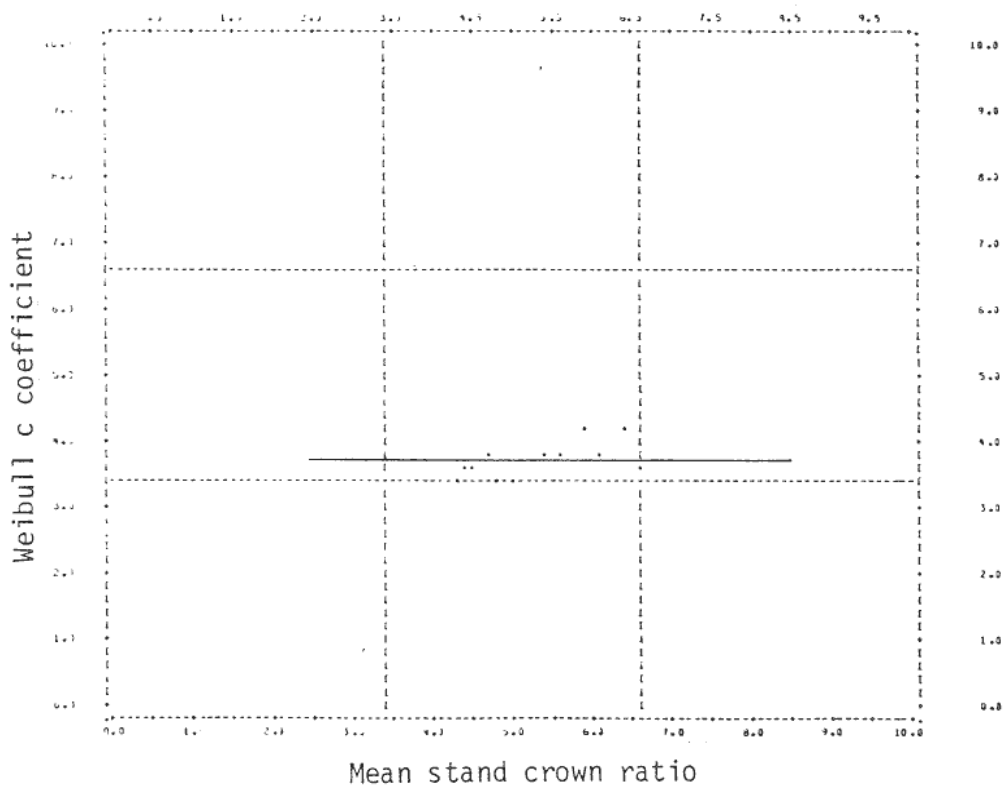


Figure 5a. Weibull shape parameter versus mean stand crown ratio for white fir in Southcentral Oregon and Northeast California.

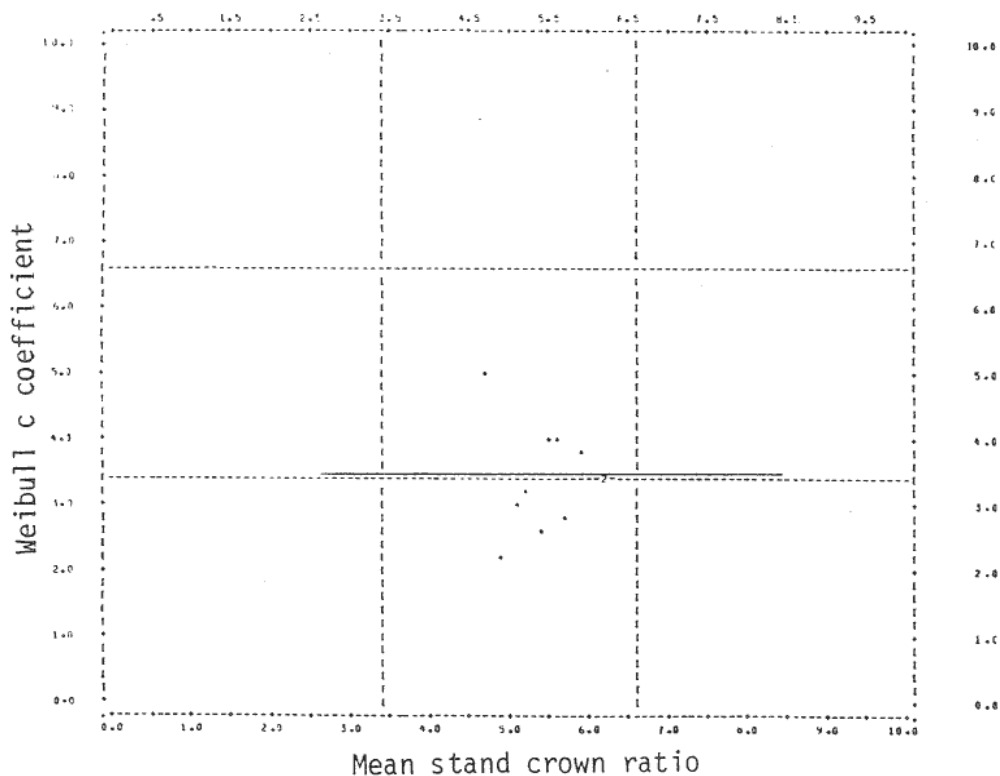


Figure 5b. Weibull shape parameter versus mean stand crown ratio for Douglas fir in Southcentral Oregon and Northeast California.

### **How it all fits together**

The first step is to estimate the mean stand crown ratio from Stand Density Index. Next, the Weibull distribution parameters are estimated from the mean stand crown ratio. Individual trees are then assigned a crown ratio from the specified Weibull distribution. At the current time, a crown ratio is picked from the Weibull distribution according to the tree's basal area percentile, but alternatives to this procedure are under study.

As the growth and yield projection continues through time, the SDI values change, as does a tree's basal area percentile. As the SDI values change, so does the Weibull distribution from which crown ratio values are drawn. The change in crown ratio from one projection cycle to the next is obtained by subtracting the crown ratios picked from the appropriate Weibull distributions. This change value is bounded to avoid drastic changes from one cycle to the next, but to date, this has not been a problem.

### **Summary and discussion**

Predicting crown ratios using Stand Density Index and the Weibull distribution appears to be a promising alternative to the traditional crown ratio estimation procedure in Prognosis. This new approach is sensitive to changing stand density, while being "well behaved" in terms of not producing widely differing crown ratios from one projection cycle to another. Two of the Weibull parameters can be estimated with species dependent constant values, while the third is easily computed from mean crown ratio.

This paper reports results of work in progress and many questions remain to be answered. Alternative ways of picking crown ratios from the specified Weibull are being examined. Also, some argument exists concerning the maximum SDI values reported in this paper, and even to the more general problem of how to determine a maximum SDI for a given species in a given area. As a result, some people may question the validity of predicting crown ratio, and subsequently the Weibull parameters, from such a controversial starting point. However, by using relative SDI in deriving the Weibull parameter estimates instead of maximum SDI, these arguments seem to lose much of their validity. Regardless of how SDI is determined, it maintains its relative position to a constant value. However, this is being examined. Other questions may arise as the procedure is applied to the wide spectrum of field applications, but preliminary results look very good.

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