Computational Design Tool for Evaluating the Stability of Large Wood Structures
Validation Calculations for Log ID: "Foot 2" in the Sample Application
(Refer to Appendix A for Model Calculation Outputs)

Constant Values

γ_w = Specific weight of water = 62.4 lb/ft^3

g = Gravitational acceleration constant = 32.2 ft/s^2

Hydrologic and Hydraulic Calculations

Meander velocity, u_m

$$u_m = u_{avg}\left[1.74 - 0.52\log\left(\frac{R_c}{W_{BF}}\right)\right] \qquad \left[for\ \frac{R_c}{W_{BF}} \leq 26\right]$$

Where:

u_{avg} = Average cross section velocity (from HEC-RAS) = 3.40 ft/s

R_c = Radius of curvature of the channel centerline = 500 ft

W_{BF} = Bankfull width = 94.0 ft

Variable	Units	Calculated Value		%
		Model	Check	Difference
u_m	ft/s	4.63	4.63	0.0000%

Stream Bed Substrate Properties Calculations

Dry Unit Weight of Substrate, γ_{bed}

$$\gamma_{soil} = 1{,}600 + 300 \log D_{50} \quad [kg/m^3]$$

Where:

D_{50} = Median grain size = 88.80 mm

And 1 $kg/m^3 \approx 0.062428\ lb/ft^3$

Variable	Units	Calculated Value		% Difference
		Model	Check	
γ_{bed}	lb/ft^3	136.4	136.4	0.0000%

Average Saturated Weight of Substrate, γ'_{bed}

$$\gamma'_{bed} = \gamma_{sat} - \gamma_w$$

Where:

γ_{sat} = Specific weight of saturated soils (with pore water weight), which is found by:

$$\gamma_{sat} = \frac{(SG_{rock} + e)\gamma_w}{1 + e}$$

Where:

SG_{rock} = Specific gravity of soils = assumed to be 2.65 for quartz particles

e = Void ratio, which is found by:

$$e = \left(\frac{SG_{rock} \times \gamma_w}{\gamma_{soil}}\right) - 1$$

Therefore:

e = Void ratio = 0.2123

γ_{sat} = Specific weight of saturated soils (with pore water weight) = 147.33 lb/ft^3

Variable	Units	Calculated Value		% Difference
		Model	Check	
γ'_{bed}	lb/ft^3	84.9	84.9	0.0000%

Wood Properties Calculations

Air-dried Timber Density (at 12% Moisture Content), γ_{Td}

$$\gamma_{Td} = SG_T \times \gamma_w \times 1.12$$

Where:

SG_T = Average oven dried specific gravity of wood for Western redcedar = 0.32 (12% moisture content basis, from Table 1A in USFS Research Note NRS-38 (2009), “Specific Gravity and Other Properties of Wood and Bark for 156 Tree Species Found in North America”)

Variable	Units	Calculated Value		% Difference
		Model	Check	
γ_{Td}	lb/ft^3	22.4	22.4	0.0000%

Green Timber Density, γ_{Tgr}

$$\gamma_{Tgr} = SG_T \times \gamma_w \times MC$$

Where:

SG_T = Average oven dried specific gravity of wood for Western redcedar = 0.31 (green volume basis, from Table 1A in USFS Research Note NRS-38 (2009))

MC = Typical moisture content of freshly sawn timber for Western redcedar = 1 + 0.40 = 1.40 (from Table 1A in USFS Research Note NRS-38 (2009))

Variable	Units	Calculated Value		% Difference
		Model	Check	
γ_{Tgr}	lb/ft^3	27.0	27.0	0.0000%

Geometry Calculations

Length of Rootwad, L_{RW}

$$L_{RW} = LF_{RW} \times D_{TS}$$

Length factor for rootwad = 1.50

D_{TS} = Diameter of the tree stem (bole) = 2.25 ft

Variable	Units	Calculated Value		%
		Model	Check	Difference
L_{RW}	ft	3.38	3.38	0.0000%

Diameter of Rootwad, D_{RW}

$$D_{RW} = DF_{RW} \times D_{TS}$$

Diameter factor for rootwad = 3.00

D_{TS} = Diameter of the tree stem (bole) = 2.25 ft

Variable	Units	Calculated Value		%
		Model	Check	Difference
D_{RW}	ft	6.75	6.75	0.0000%

Volume of Tree Stem, V_{TS}

$$V_{TS} = \pi \left(\frac{D_{TS}}{2}\right)^2 L_{TS}$$

Where:

D_{TS} = Diameter of the tree stem (bole) = 2.25 ft

L_{TS} = Length of the tree stem (not including rootwad), which is found by:

$$L_{TS} = L_T - L_{RW}$$

Where:

L_T = Total length of the tree (including rootwad) = 32.0 ft

L_{RW} = Length of rootwad = 3.38 ft

Therefore:

L_{TS} = Length of the tree stem (not including rootwad) = 28.62 ft

Variable	Units	Calculated Value		%
		Model	Check	Difference
V_{TS}	ft^3	113.8	113.8	0.0000%

Volume of Tree Stem below Thalweg Elevation, $V_{TS\downarrow Thw}$

The complex geometric equations in the model were verified using an online calculator found at: http://planetcalc.com/1442/

Variable	Units	Calculated Value		%
		Model	Check	Difference
$V_{TS\downarrow Thw}$	ft^3	0.6	0.6	0.0000%

Volume of Rootwad, V_{RW}

$$V_{RW} = \frac{\pi L_{RW}}{3}\left[\left(\frac{D_{RW}}{2}\right)^2 + \left(\frac{D_{RW}}{2}\right)\left(\frac{D_{TS}}{2}\right) + \left(\frac{D_{TS}}{2}\right)^2\right](1-\eta)$$

Where:

D_{TS} = Diameter of the tree stem (bole) = 2.25 ft

L_{RW} = Length of the rootwad = 3.38 ft

D_{RW} = Diameter of the rootwad = 6.75 ft

η = Rootwad porosity = 0.20

Variable	Units	Calculated Value		%
		Model	Check	Difference
V_{RW}	ft^3	46.5	46.6	0.2151%

Volume of Roowtad below Thalweg Elevation, $V_{TS\downarrow Thalweg}$

The computational model approximates the volume of the rootwad as a tilted frustum. The volume below the thalweg elevation is found by using the slice method to break the rootwad into 30 discs. The volume below the thalweg is approximated for each disc, and then added together to approximate the total volume of the rootwad below the thalweg elevation. The calculations are too complex to show here, but each individual step was checked by a calculator as the model was built.

Variable	Units	Calculated Value		%
		Model	Check	Difference
$V_{RW\downarrow Thw}$	ft^3	10.8	N/A	N/A

Projected Area of Wood, A_{Tp}

The computational model approximates the projected area of the tree stem and rootwad separately, and then adds them together. The projected area of the tree stem is found by using the partial area method to break the stem up into 200 evenly sized discs. Each disc was then evaluated to determine how much area is exposed to flow. The projected area for the rootwad calculations uses a complex geometric relationship to determine the exposed area to flow. The calculations are too complex to include in this write up, but each individual step was checked by a calculator as the model was built. The calculations for the projected area of the log are summarized in Cells (S75:U81) of the "Single Log Design" worksheet. The calculations and formulas for the projected area of the rootwad are shown in the Cells (W75:AE80) of the "Single Log Design" worksheet.

Variable	Units	Calculated Value		% Difference
		Model	Check	
A_{Tp}	ft^2	11.47	N/A	N/A

Embedded Length of Log, $L_{T,em}$

The embedded length of the log was calculated by two methods in the spreadsheet. The first method used complex geometric equations which can be found in Cells (S45:AB53) of the "Single Log Design" worksheet. The second approach used the partial area method to approximate the length of the embedment, with the output shown in Cell AY43. Both methods produce very similar outputs, indicating that the calculation is accurate. The complex geometric equations are considered to be slightly more accurate (by definition, the partial area method is an approximation), so the geometric equations are referenced for the embedded length in the design.

Variable	Units	Calculated Value		% Difference
		Model	Check	
$L_{T,em}$	ft	21.02	21.04	0.0951%

Depths of Burial and Soil Volumes, $d_{b,max}$, $d_{b,avg}$, V_{dry}, *and* V_{sat}

The model uses the partial area method to divide the log into 200 even sections and then evaluate the depths of burial (maximum and average) and soil overburden volume (dry and saturated) for each disc. The model then uses geometric relationships between the top of the log, the ground surface, the position within the channel (channel bed or bank), and the water surface elevation to determine these parameters. Each formula was checked during development, but these calculations are too complex to include in this paper. However, the model outputs were compared to the cross section plot, and the model appears to provide reasonable results. As a second check, the following approximation was used (note – since all of the soil was below the 50-year flood stage, $V_{dry} = 0$):

$$V_{soil} = V_{sat,bed} + V_{sat,bank} = \left[L_{T,em} D_{TS} d_{b,avg}\right]_{bed} + \left[L_{T,em} D_{TS} d_{b,avg}\right]_{bank}$$

Where:

$L_{T,em}$ = Embedded length of the tree = 5.69 ft in bed, 15.33 ft in bank

D_{TS} = Diameter of the tree stem (bole) = 2.25 ft

$d_{b,avg,\ bed}$ = Average burial depth to crown of the log = 0.32 ft in bed, 1.70 ft in bank

Variable	Units	Calculated Value		%
		Model	Check	Difference
V_{dry}	ft^3	0.0	0.0	0.0000%
$V_{sat, bed}$	ft^3	4.1	4.1	0.0000%
$V_{sat, bank}$	ft^3	58.1	58.6	0.8606%

Vertical Force Analysis Calculations

Net Buoyancy Force, F_B

The buoyant force, F_B, acting on the structure is equal to the weight of displaced water:

$$F_B = \gamma_W V_{T\downarrow WSE}$$

Where:

$V_{T\downarrow WSE}$ = Volume of tree below the water surface elevation, WSE = 160.3 ft^3

Variable	Units	Calculated Value		%
		Model	Check	Difference
F_B	lbf	10,005	10,003	-0.0200%

Gravity Force, W_T

$$W_T = \gamma_{Td} V_{Td} + \gamma_{Tgr} V_{Tgr}$$

Where:

γ_{Td} = Specific dry weight of Douglas-fir, Interior West = 34.9 lb/ft^3

V_{Td} = Volume of tree above channel thalweg = 148.9 ft^3

γ_{Tgr} = Specific green weight of tree = 39.0 lb/ft^3

V_{Tgr} = Volume of tree below channel thalweg = 0.6 + 10.8 = 11.4 ft^3

Variable	Units	Calculated Value		%
		Model	Check	Difference
W_T	lbf	5,649	5,641	-0.1416%

Soil Ballast Force, F_{soil}

All of the soil is below the 50-year flood stage ($V_{soil,dry}$ = 0 ft^3), so the following simplified equation applies:

$$F_{soil} = \left[V_{soil,sat}\ \gamma_s'\right]_{bed} + \left[V_{soil,sat}\ \gamma_s'\right]_{bank}$$

Where:

$V_{soil,sat}$ = Volume of saturated soil below the water surface elevation WSE = 4.1 ft^3 for the channel bed substrate and 58.1 ft^3 for the bank soils

γ_s' = Effective buoyant (saturated) unit weight of soil = 84.9 lb/ft^3 for the channel bed substrate and 85.3 lb/ft^3 for the bank soils

Variable	Units	Calculated Value		%
		Model	Check	Difference
F_{soil}	lbf	5,308	5,304	-0.0754%

<u>Lift Force,</u> F_L

The model only calculates a lift force if the bottom of the log is exposed to flow. The log identified as “Foot #2” does not meet this criteria, so the lift force equals 0.

Variable	Units	Calculated Value		%
		Model	Check	Difference
F_L	lbf	0	0	0.0000%

<u>Vertical Interaction Forces with Adjacent Logs,</u> $F_{W,V}$

Checking the adjacent log forces is beyond the scope of this validation. However, the total resulting vertical forces acting on the “Foot #2” log were added together to verify the model value.

Variable	Units	Calculated Value		%
		Model	Check	Difference
$F_{W,V}$	lbf	9,857	9,858	0.0101%

<u>Vertical Anchoring Forces (Boulders),</u> $F_{A,Vr}$

Three boulder anchors are proposed in the design. One is a 4.75-foot diameter ballast boulder (placed on top of log). The other two are 5-foot diameter deadman boulders. The vertical resisting force is found by the following equation:

$$F_{A,Vr} = W_r - F_{L,r}$$

Where:

W_r = weight of the boulder, which is found by:

$$W_r = V_{r,dry}\gamma_{rock} + V_{r,wet}(\gamma_{rock} - \gamma_w)$$

Where:

$V_{r,dry}$ = Volume of rock above the water surface elevation, WSE = 0 ft^3

$V_{r,wet}$ = Volume of rock submerged below the water surface elevation, WSE = 56.1 ft^3 for the ballast boulder and 65.4 ft^3 for the deadman boulders

γ_{rock} = Specific weight of rock = 165 lb/ft^3

The portion of the single ballast boulder that is exposed to flow will be subject to lift forces. The lift force on the boulder, $F_{L,r}$, is found by:

$$F_{L,r} = \frac{C_{Lrock}A_{Pr}\gamma_w u_{des}^2}{2g}$$

Where:

C_{Lrock} = Lift coefficient for large roughness elements = 0.17 (D’Aoust and Millar, 2000)

A_{Pr} = Projected area of rock in the plane perpendicular to flow = 12.29 ft^2

u_{des} = Design velocity of flow = 4.63 ft/s

The anchor force for the boulder ballast was validated as follows:

Variable	Units	Calculated Value		%
		Model	Check	Difference
$F_{A,Vr}$	lbf	5,714	5,712	-0.0350%

The anchor force for each of the deadman boulders were validated as follows:

Variable	Units	Calculated Value		%
		Model	Check	Difference
$F_{A,Vr}$	lbf	6,715	6,710	-0.0745%

Vertical Force Balance, ΣF_V

$$\sum F_V = (W_T + F_{soil} + F_{A,V}) - (F_B + F_L + F_{W,V})$$

Variable	Units	Calculated Value		%
		Model	Check	Difference
ΣF_V	lbf	10,239	10,233	-0.0586%

Vertical Factor of Safety, FS_V

$$FS_V = \frac{W_T + F_{soil} + F_{A,V}}{F_B + F_L + F_{W,V}}$$

Variable	Units	Calculated Value		%
		Model	Check	Difference
FS_V	-	1.52	1.52	0.0000%

Horizontal Force Analysis Calculations

Drag Forces, F_D

$$F_D = \frac{C_D^* A_{Tp} \gamma_w u_{des}^2}{2g}$$

A_{Tp} = Projected area of large wood in the plane perpendicular to flow = 11.47 ft^2

u_{des} = Design velocity of flow = 4.63 ft/s

C_D^* = Effective drag coefficient for a submerged tree, which is equal to:

$$C_D^* = 0.997(C_{Di} + C_w)\left(1 - \frac{A_{Tp}}{A_W}\right)^{-2.06}$$

Where:

C_{Di} = Drag coefficient of wood in a flow of infinite extent (no boundary) = 0.99

A_W = Wetted area of channel at design discharge = 1,650 ft^2

C_W = Wave drag coefficient of wood, which is found by:

$$C_w = \frac{\pi^2}{32} Fr_L^{-6} exp\left(-\left(z_{T,CL\downarrow WSE} / D_{TS}\right)/2Fr_L^2\right)$$

Where:

$z_{T,CL\downarrow WSE}$ = Distance to large wood centerline from the water surface elevation = 7.38 ft

Fr_L = Log Froude number, which is found by:

$$Fr_L = \frac{u_{des}}{\sqrt{gD_{TS}}}$$

Variable	Units	Calculated Value		%
		Model	Check	Difference
Fr_L	-	0.54	0.54	0.0000%
C_w	-	0.05	0.05	0.0000%
C_{D*}	-	1.04	1.05	0.7692%
F_D	lbf	249	248	-0.4016%

Friction Force, F_F

$$F_F = \mu_{bed} F_N$$

Where:

F_N = Normal force of soil on structure = 10,239 lbf (from horizontal force balance)

μ_{bed} = Coefficient of friction, which is assumed to equal:

$$\mu_{bed} = tan\phi$$

Where:

ϕ = Internal friction angle of soils (stream substrate or bank soils) = 41 deg

Therefore:

μ_{bed} = 0.87

Variable	Units	Calculated Value		% Difference
		Model	Check	
F_F	lbf	8,901	8,908	0.0786%

Passive Soil Pressure Force, F_P

$$F_P = 0.5 K_p F_{soil}$$

Where:

F_{soil} = Soil ballast force = 5,308 lbf (from horizontal force balance)

K_P = Coefficient of passive earth pressure, which is given by:

$$K_P = \tan^2\left(45 + \frac{\phi}{2}\right)$$

Where:

ϕ = Internal friction angle of soils (stream substrate or bank soils) = 41 deg

Variable	Units	Calculated Value		% Difference
		Model	Check	
K_P	-	4.81	4.81	0.0000%
F_P	lbf	12,779	12,779	0.0000%

Horizontal Interaction Forces with Adjacent Logs, $F_{W,H}$

Checking the adjacent log forces is beyond the scope of this validation. However, the total resulting horizontal forces acting on the “Foot #2” log were added together to verify the model value.

Variable	Units	Calculated Value		% Difference
		Model	Check	
$F_{W,H}$	lbf	294	294	0.0000%

Horizontal Force Balance, ΣF_H

$$\sum F_H = \left(F_F + F_P + F_{W,H}\right) - F_D$$

Variable	Units	Calculated Value		% Difference
		Model	Check	
ΣF_H	lbf	21,724	21,725	0.0046%

Horizontal Factor of Safety, FS_H

$$FS_H = \frac{F_F + F_P + F_{W,H}}{F_D}$$

Variable	Units	Calculated Value		% Difference
		Model	Check	
FS_H	-	88.10	88.25	0.1703%

Moment Force Analysis Calculations

Driving Moment, M_D

The driving moment, M_D, about the buried stem tip of the embedded log is given by:

$$M_D = \left[F_B c_{T,B} + F_L c_L + F_D c_D + \sum F_{W,driving} c_{WI}\right] \cos\beta$$

Where:

$c_{T,B}$ = Centroid of the buoyancy force along log axis

c_L = Centroid of the lift force along log axis

c_D = Centroid of the drag force along log axis

c_{WI} = Centroid of the driving interaction forces with adjacent members along log axis

β = Tilt angle from stem tip to vertical = 0 deg

Variable	Units	Calculated Value		% Difference
		Model	Check	
$c_{T,B}$	ft	19.2	19.2	0.0000%
c_L	ft	0.0	0.0	0.0000%
c_D	ft	26.6	26.5	-0.3759%
c_{WI}	ft	Input	Input	0.0000%
M_D	lbf	324,748	324,808	0.0185%

Resisting Moment, M_R

$$M_R = \left[W_T c_{T,W} + F_{soil} c_{soil} + (F_F + F_N) c_{F,N} + F_P c_P + \sum (F_{A,H} + F_{A,V}) c_A + \sum F_{W,resisting} c_{WI}\right] \cos\beta$$

Where:

$c_{T,W}$ = Centroid of the log volume along log axis

c_{soil} = Centroid of the vertical soil forces along log axis

$c_{F\&N}$ = Centroid of friction and normal forces along log axis

c_P = Centroid of the passive soil force along log axis

c_A = Centroid of each anchor at the log axis

c_{WI} = Centroid of the resisting interaction forces with adjacent members along log axis

β = Tilt angle from stem tip to vertical = 0 deg

Variable	Units	Calculated Value		%
		Model	Check	Difference
$c_{T,W}$	ft	19.2	19.2	0.0000%
c_{soil}	ft	10.5	10.5	0.0000%
$c_{F\&N}$	ft	16.0	16.0	0.0000%
c_{P}	ft	14.0	14.0	0.0000%
c_{A}	ft	Input	Input	0.0000%
c_{WI}	ft	Input	Input	0.0000%
M_{D}	lbf	996,160	996,649	0.0491%

Moment Factor of Safety, FS_M

$$FS_M = \frac{M_R}{M_D}$$

Variable	Units	Calculated Value		%
		Model	Check	Difference
FS_{M}	-	3.07	3.07	0.0000%