WHITE PAPER

USDA Forest Service | Pacific Northwest Region | Umatilla National Forest

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Using Mathematics in Forestry: An Environmental Education Activity[1]

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INTRODUCTION

Most jobs require good mathematics skills, and forestry is no exception. Exercises in this handout will show you some ways that foresters use mathematics in their work.

Foresters usually deal with large land areas. They need to know what kind of trees grow on their area, how many of them there are, and how big they are. Measuring every tree on a national forest covering one and half million acres, and containing over 500 million trees, would not only be time consuming and expensive, but probably impossible!

For these and other reasons, foresters only measure part of a forest, a process called sampling. They assume that a measured (sampled) portion represents a whole area. Data collected from a sampled portion is expanded to provide information for an unsampled portion.

Suppose we need to know how much usable timber is present in a 100-acre ponderosa pine stand. The entire stand (all 100 acres) is our population, but we can't measure every tree because it would take too much time. So, an efficient way to inventory the ponderosa pine timber is to sample it, which we'll do by establishing plots.

[1] White papers are internal reports; they receive only limited review. Viewpoints expressed in this paper are those of the author – they may not represent positions of USDA Forest Service.
[2] This paper was prepared in 1990 for Mr. Kevin Steinmetz's sixth grade class at Humbolt Elementary School in John Day, Oregon. It has been used many times since then with other school groups.

Statistics indicate that 20 plots of ¼-acre each will provide reliable information about the ponderosa pine timber. How can we figure out a plot size? By using some mathematical formulas, of course! Previous sampling experience shows that circular plots are easier to use than square ones.

To lay out a plot in the shape of a circle, we must know its radius, which we'll figure out by using this formula:

$$\text{Area} = \text{Pi} \times \text{Radius}^2$$

> <u>Note</u>: if you don't know what pi is, here is a short description. Pi is the number you get when you divide the distance around a circle (its circumference) by the distance through its middle (its diameter). The distance around the outside of every circle is about three times the distance across it. But it's the "about" part that creates the puzzle of pi. Mathematicians call pi an irrational number because when you divide a circle's circumference by its diameter, the answer comes out in decimals that go on <u>forever</u> without any apparent pattern. Pi begins as 3.14159265, but it never ends. In 1999, a Japanese scientist used a supercomputer to calculate pi to about 206 *billion* digits, and it still goes on from there. All those digits aren't really necessary to use pi, of course – using only the first ten decimals, you can measure Earth's circumference to within a fraction of an inch. Pi is often shown in textbooks or in formulas by using this Greek symbol: π

The total area of an acre is 43,560 square feet. Since we want to use a ¼-acre plot, its area is 43,560 ÷ 4 or 10,890 square feet. We now have enough information to use the formula above to figure out our plot radius.

$$\begin{aligned}
\text{Area} &= \text{Pi} \times \text{Radius}^2 \\
10{,}890 \text{ square feet} &= 3.1416 \times \text{Radius}^2 \\
10{,}890 \text{ square feet} \div 3.1416 &= \text{Radius}^2 \\
3{,}466.39 \text{ square feet} &= \text{Radius}^2 \\
\sqrt{3{,}466.39} \text{ square feet} &= \text{Radius} \\
58.9 \text{ feet} &= \text{Radius}
\end{aligned}$$

Each ¼-acre plot will be a circle with a radius of 58.9 feet. We'll use a wooden stake as a plot center, and mark plot boundaries by measuring out a radius in several directions from the plot-center stake.

Great! We've just used our knowledge about geometry of a circle to design plots that will help us answer our inventory question about ponderosa pine timber.

Exercise 1: If our circular plot has a radius of 58.9 feet, what is its diameter? [Do you remember the relationship between a circle's radius and its diameter?]

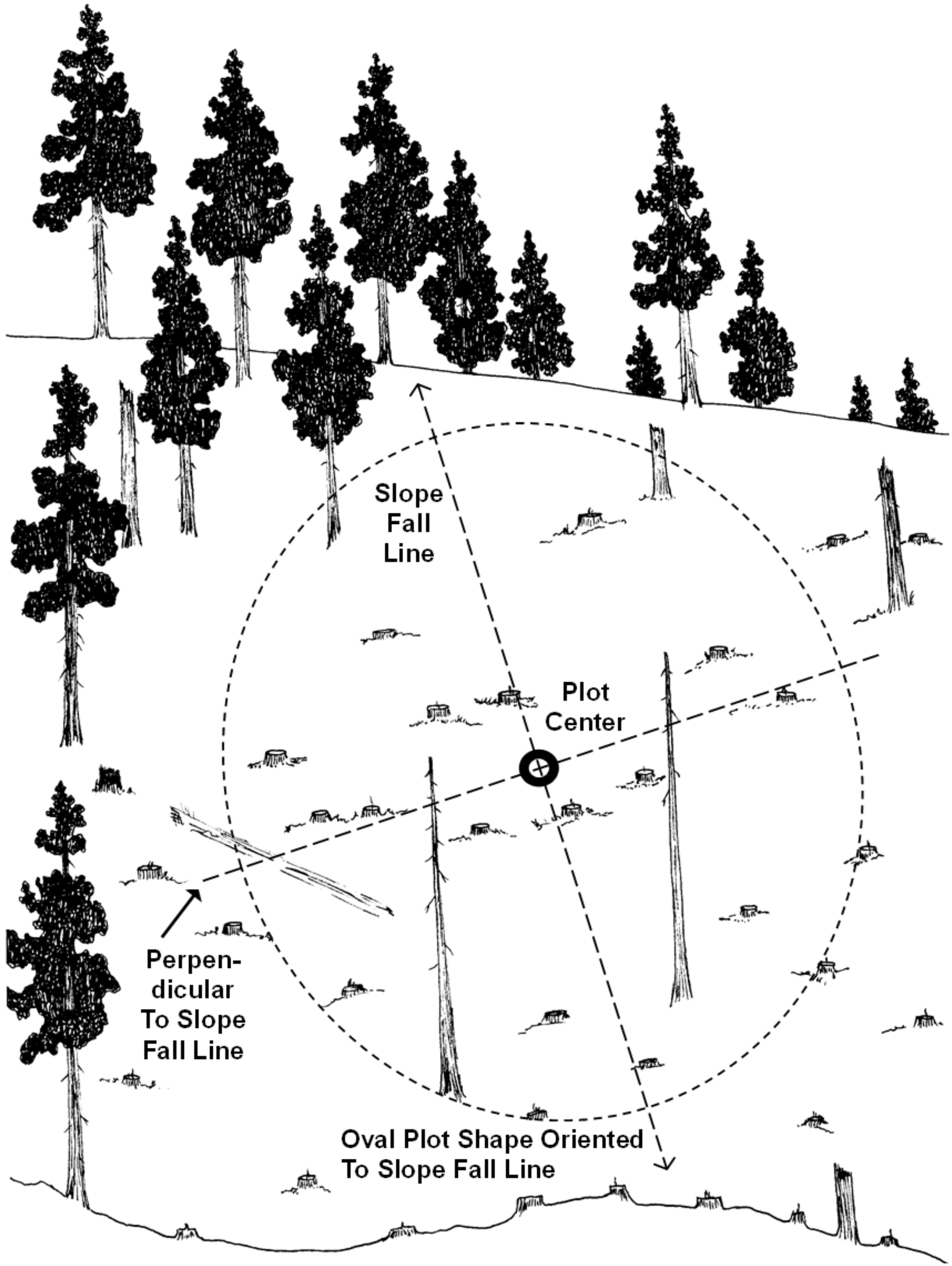
Diameter is: ____________________ feet.

Exercise 2: What is the area (in square feet) and the radius (in feet) for circular plots of the following sizes? [Don't forget that 1 acre contains 43,560 square feet.]

Plot Size (Acres)	**Plot Area (Square Feet)**	**Plot Radius (Feet)**
1/4	10,890	58.9
1/5	____________	____________
1/10	____________	____________
1/20	____________	____________
1/50	____________	____________
1/100	____________	____________
1/250	____________	____________
1/300	____________	____________
1/500	____________	____________
1/1000	____________	____________

If the ground surface near a plot center is absolutely flat, then a plot is a perfect circle with a radius as it was calculated above. But problems can start when you are working on a hill because a plot's radius then depends on a hill's steepness – plot radius is only slightly longer for hills that are not steep, but they can be quite a bit longer for very steep hills.

By projecting a plot's radius on a steep hillside, as is shown in the diagram on the next page, we see that it becomes oval in shape, not circular.



On sloping ground, plots have an oval shape with their long axis parallel to the slope (called 'slope fall line'). Note that a line perpendicular to the slope forms a 'right angle' with the slope fall line. Plots on sloping ground need to have their radius adjusted by using a factor that converts slope distance to what is called <u>horizontal distance</u>.

How do we adjust plot radius for a sample plot occurring on sloping ground? Well, you could figure out what an adjusted radius should be by using trigonometry (secants), but most foresters just carry around something called a slope correction table.

Here is part of the slope correction table for slopes ranging up to 61 percent:

Table 1: Slope correction factors

Slope Percent	Correction Factor
0 – 9	1.00
10 – 17	1.01
18 – 22	1.02
23 – 26	1.03
27 – 30	1.04
31 – 33	1.05
34 – 36	1.06
37 – 39	1.07
40 – 42	1.08
43 – 44	1.09
45 – 47	1.10
48 – 49	1.11
50 – 51	1.12
52 – 53	1.13
54 – 55	1.14
56 – 57	1.15
58 – 59	1.16
60 – 61	1.17

Now, let's use table 1's slope correction factors to figure out if some trees near our plot edge are "in or out" (inside or outside of the plot radius). Trees near a plot edge are referred to as *borderline* trees. Here is a way to measure borderline trees:

1. Use an instrument called a clinometer to measure slope percent from the center (side) of a borderline tree to a plot center (let's say that it is 30 percent).
2. Find a slope correction factor in table 1 for the slope percent you just measured (correction factor is 1.04 for 30 percent).
3. Multiply a correction factor by plot radius. This is called a corrected radius. For our ¼-acre plots, the result is: 58.9 feet × 1.04 = 61.26 feet.

4. Measure slope distance from center (side) of a borderline tree to plot center. If measured distance is less than the corrected radius (61.26 feet in this example), then the tree is in a plot; if measured distance is more than the corrected radius, the tree is out of a plot.

   Exercise 3: Are these trees near a ¼-acre plot in or out? Record slope correction factor (from table 1) and corrected plot radius for each tree too.

Tree Number	**Slope Percent From Tree to Plot Center**	**Slope Correction Factor**	**Corrected Plot Radius (Feet)**	**Distance From Tree to Plot Center (Feet)**	**In or Out?**
1	20	___________	___________	59.1	_______
2	25	___________	___________	58.6	_______
3	30	1.04	61.26	62.9	Out
4	35	___________	___________	62.4	_______
5	45	___________	___________	63.7	_______
6	50	___________	___________	59.9	_______
7	60	___________	___________	60.3	_______

MEASURING CIRCUMFERENCE AND DIAMETER

Now that we know how to figure out which trees are in or out of a plot, we need to learn how to measure the size of trees that are on a plot. First, we need to find out how big around each tree is.

There are two main ways we can describe the size of round objects like tree stems:

- We can measure their circumference, which is the distance around the outside of the trunk, or
- We can measure circumference and convert it to diameter, which is the distance through the middle of a tree's trunk.

Foresters use special measuring tapes that show a tree's circumference on one side, and its equivalent diameter on the other side. How is that done? Actually, it's easy to do because circumference and diameter are closely related:

$$\text{Circumference} = \text{Pi} \times \text{Diameter}$$

$$\text{Diameter} = \text{Circumference} \div \text{Pi}$$

[By now, have you noticed that most mathematical equations pertaining to circles use pi?]

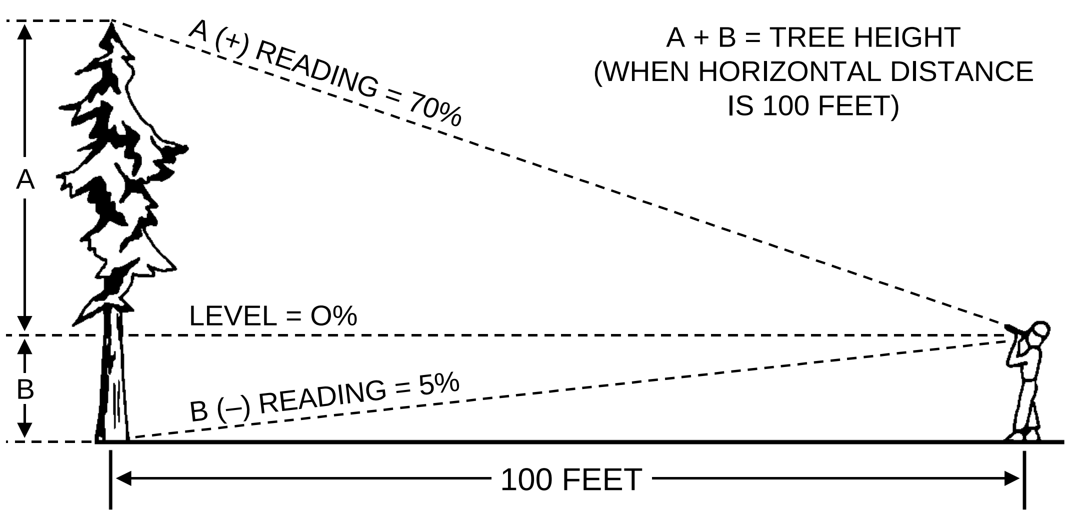
Exercise 4: What are diameters or circumferences of the following trees? (You might be interested to know that these are actual measurements for some of the biggest trees found in the Blue Mountains of northeastern Oregon.)

Species	Circumference (Inches)	Diameter (Inches)
Subalpine Fir	132	__________
Western Juniper	151	__________
Lodgepole Pine	__________	34.7
Ponderosa Pine	242	__________
Engelmann Spruce	__________	63.3
Grand Fir	218	__________
Quaking Aspen	129	__________
Douglas-fir	__________	65.9

MEASURING TREE HEIGHTS

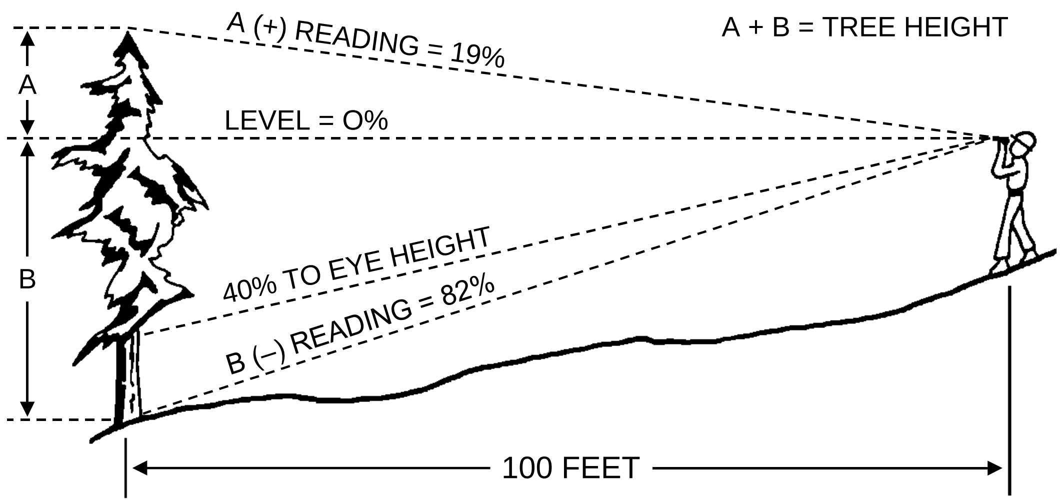
After measuring a tree's diameter or circumference, we then need to find out how tall it is. This can be tricky until you get the hang of it, especially when working on hillsides or sloping ground.

Foresters measure heights using an instrument called a clinometer (short for an "incline meter"). Many clinometers are based on percent, which means they are designed to be used at a distance of 100 feet from the tree. Let's look at an example on flat ground:



The A (+) readings on a clinometer mean that you are looking up; the B (–) readings mean you are looking down. Level (not up or down) readings are 0 on a clinometer. In our example above, the A reading (70) was looking up at the top of a tree; the B reading (5) was looking down at its base (where its trunk meets the ground). To get this tree's total height, you add the top (70) and bottom (5) readings together: 70 + 5 = 75 feet tall for our example tree.

Now, let's look at a situation where measuring tree height is a little more complicated. When using a clinometer on sloping ground (hillsides), you must apply a slope correction factor just like we did when checking whether borderline trees were in or out of the plot area. Here's a height measurement example for sloping ground:



Here is the process you'd use to measure this tree's height:

1. Use the clinometer to measure the slope percent from you to your 'eye height' on the tree trunk (this sighting is a line that's parallel to the ground surface). In our example, the 'eye height' slope percent is 40.
2. Find a slope correction factor in table 1 for the slope percent you just measured (1.08 is the slope correction factor for 40 percent).

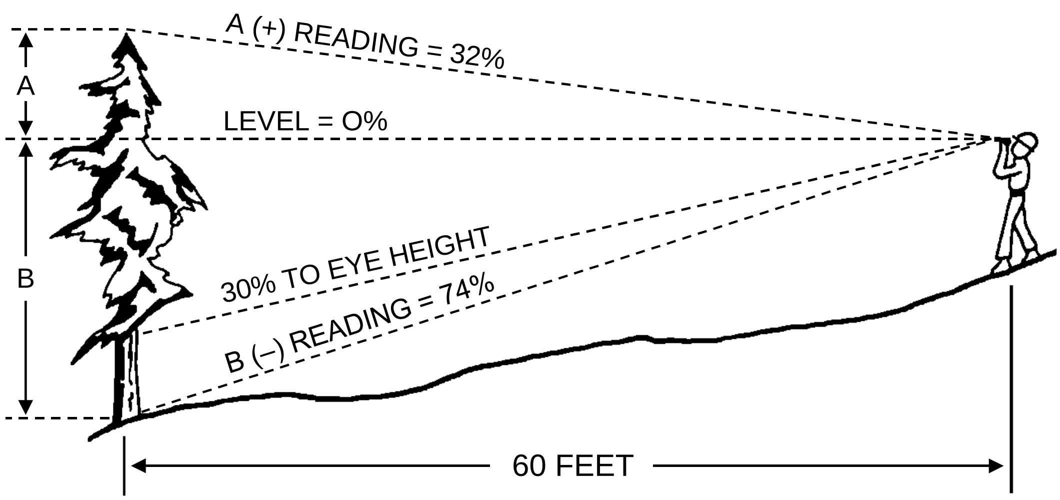
> Note: many clinometers show slope correction factors on the same scale as the slope percent values, which is handy because then you don't have to look them up in a table.

3. Multiply the slope correction factor by the baseline distance. In our example with a 100-foot baseline, the result is: 100 feet × 1.08 = 108 feet. This means that on a 40 percent slope, you must be 108 feet away from a tree to get the same answer as if you were 100 feet away on flat (level) ground.
4. *Back up to 108 feet from the tree* before taking clinometer readings of its top and base.
5. Take clinometer readings of the tree's top and base, and add them together to come up with a total height. In our example, the result is: 19 + 82 = 101 feet.

Exercise 5: What are the heights of these trees?

Tree	Slope Percent to Tree	Slope Correction Factor	Corrected Baseline (Feet)	Clinometer Readings: Up (A)	Clinometer Readings: Down (B)	Total Height (Feet)
1	35	________	________	96	32	_______
2	40	1.08	108	19	82	101
3	45	________	________	43	8	_______
4	50	________	________	78	16	_______
5	55	________	________	91	27	_______
6	60	________	________	66	35	_______

Next we'll look at another situation where measuring tree height is more complicated than usual. It is often hard to measure heights from 100 feet away if thick brush or downed logs hide your view of a tree's base. In these cases, you'll need to measure height from someplace other than 100 feet away and adjust clinometer results accordingly. Here is our example tree:



Here is what you'd do to measure this tree's height:

1. Use a clinometer to measure slope percent to eye height on a tree (30 percent in this exam-ple).
2. Get a slope correction factor from table 1 (1.04 for 30 percent).
3. Multiply the slope correction factor by your baseline distance: 60 feet × 1.04 = 62.4 feet.
4. *Back up to 62.4 feet* before taking clinometer readings of a tree's top and base.

5. Take clinometer readings of the top and base, and add them together. In our example, the result is: 32 + 74 = 106.

   Does this mean that the tree is 106 feet tall? No, it does not! Do you remember that a clinometer is designed to be used with a 100-foot baseline (see page 7)? Any time that you are not 100 feet away from a tree (slope-corrected distance), clinometer readings must be adjusted to account for a different baseline distance.
   In our example, the slope-corrected baseline distance was only 60 feet, not 100.

6. Calculate a baseline adjustment factor by dividing your baseline distance by 100:
   60 feet ÷ 100 = 0.6.

7. Multiply the sum of clinometer readings (see step 5 above) by the baseline adjustment factor to finally get a total height for your tree: 106 × 0.6 = 63.6 feet.

Exercise 6: What are the heights of these trees?

Tree	Baseline Distance (Feet)	Baseline Adjustment Factor	Clinometer Readings: Up (A)	Clinometer Readings: Down (B)	Total Height (Feet)
1	40	________	89	47	________
2	50	________	73	26	________
3	60	.6	32	74	63.6
4	65	________	68	29	________
5	75	________	52	11	________
6	90	________	64	26	________

Now, let's pull some of our information together. We began sampling the 100-acre ponderosa pine stand last week and three plots are done so far. Here is the sample data at this point:

PLOT 1 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	32.6	109	196
2	Ponderosa Pine	27.4	101	173
3	Grand Fir	18.9	88	93
4	Douglas-fir	29.4	106	204
5	Grand Fir	12.3	72	79
6	Quaking Aspen	8.6	66	58

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
7	Western Larch	14.2	91	87
8	Ponderosa Pine	9.6	48	54
9	Ponderosa Pine	15.2	68	74
10	Western Larch	16.9	96	86
	Average	________	________	________

How many trees per acre? ________

PLOT 2 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	33.9	111	194
2	Ponderosa Pine	30.7	108	188
3	Douglas-fir	26.9	98	147
4	Ponderosa Pine	27.8	102	156
5	Douglas-fir	21.6	92	106
6	Ponderosa Pine	28.9	103	129
	Average	________	________	________

How many trees per acre? ________

PLOT 3 (¼ acre)

Tree	Species	Diameter (Inches)	Height (Feet)	Age (Years)
1	Ponderosa Pine	26.2	98	114
2	Grand Fir	22.9	95	102
3	Ponderosa Pine	36.8	109	149
4	Douglas-fir	42.9	113	196
5	Lodgepole Pine	12.6	74	82
6	Ponderosa Pine	28.4	100	121
7	Ponderosa Pine	22.9	91	106
8	Grand Fir	18.7	89	97
9	Douglas-fir	27.6	99	112
	Average	________	________	________

How many trees per acre? ________

<u>Exercise 7</u>: Here's what you need to do now:

1. Calculate average diameter, height, and age for each of the 3 plots, and record it in the tables (calculate an arithmetic average by summing the values in each column and then dividing by the total number of sample trees).
2. Calculate the number of trees per acre that each plot represents, and record your answers in the tables.

   How can you do this? Remember that each plot samples exactly ¼ of an acre. If you count the number of trees that are "in" on your plot, and then multiply by 4 to expand the sample to a whole acre, you will know how many trees per acre are represented by your sample.

> <u>Note</u>: when you sample (measure) a fraction of an acre, and then later want to expand the sample data so it represents a whole acre, the sample values must be multiplied by the denominator of plot size. For a 1/5-acre plot, each sample tree represents 5 trees (the denominator value); for a 1/20-acre plot, each sample tree represents 20 trees; and so forth.

3. Calculate an average, for the 3 plots combined, by filling in the table below:

Plot	**Average Diameter (Inches)**	**Average Height (Feet)**	**Average Age (Years)**	**Trees Per Acre**
1	_________	_________	_________	_________
2	_________	_________	_________	_________
3	_________	_________	_________	_________
Stand Average	_________	_________	_________	_________

4. What proportion of the sample trees, on the 3 plots combined, were ponderosa pines (don't forget that we are interested in how much ponderosa pine timber we have on the 100-acre tract)? __________ percent.
5. Fill in the blanks in this summary of our results so far. "After completing three plots, we can now say that the sampled area has an average of __________ trees per acre, with an average diameter of __________ inches, an average height of __________ feet, and an average age of __________ years. The proportion of ponderosa pines on the 3 plots was __________ percent."

Since this summary is for 3 plots only, the data will almost certainly change after all 20 samples are completed!

LET'S SUMMARIZE

What have you learned from this exercise?

- How to figure out the radius of a circular plot after being given its area.
- How to adjust a plot radius to account for sloping ground.
- That many mathematical formulas involving circles use pi, a special constant that we round off to 3.1416.
- How to measure circumference and diameter of trees.
- How to use a clinometer to measure tree heights on either flat or sloping ground.

APPENDIX: SILVICULTURE WHITE PAPERS

White papers are internal reports, and they are produced with a consistent formatting and numbering scheme – all papers dealing with Silviculture, for example, are placed in a silviculture series (Silv) and numbered sequentially. Generally, white papers receive only limited review and, in some instances pertaining to highly technical or narrowly focused topics, the papers may receive no technical peer review at all. For papers that receive no review, the viewpoints and perspectives expressed in the paper are those of the author only, and do not necessarily represent agency positions of the Umatilla National Forest or the USDA Forest Service.

Large or important papers, such as two papers discussing active management considerations for dry and moist forests (white papers Silv-4 and Silv-7, respectively), receive extensive review comparable to what would occur for a research station general technical report (but they don't receive blind peer review, a process often used for journal articles).

White papers are designed to address a variety of objectives:

(1) They guide how a methodology, model, or procedure is used by practitioners on the Umatilla National Forest (to ensure consistency from one unit, or project, to another).

(2) Papers are often prepared to address ongoing and recurring needs; some papers have existed for more than 20 years and still receive high use, indicating that the need (or issue) has long standing – an example is white paper #1 describing the Forest's big-tree program, which has operated continuously for 25 years.

(3) Papers are sometimes prepared to address emerging or controversial issues, such as management of moist forests, elk thermal cover, or aspen forest in the Blue Mountains. These papers help establish a foundation of relevant literature, concepts, and principles that continuously evolve as an issue matures, and hence they may experience many iterations through time. [But also note that some papers have not changed since their initial development, in which case they reflect historical concepts or procedures.]

(4) Papers synthesize science viewed as particularly relevant to geographical and management contexts for the Umatilla National Forest. This is considered to be the Forest's self-selected 'best available science' (BAS), realizing that non-agency commenters would generally have a different conception of what constitutes BAS – like beauty, BAS is in the eye of the beholder.

(5) The objective of some papers is to locate and summarize the science germane to a particular topic or issue, including obscure sources such as master's theses or Ph.D. dissertations. In other instances, a paper may be designed to wade through an overwhelming amount of published science (dry-forest management), and then synthesize sources viewed as being most relevant to a local context.

(6) White papers function as a citable literature source for methodologies, models, and procedures used during environmental analysis – by citing a white paper, specialist reports can include less verbiage describing analytical databases, techniques, and so forth, some of which change little (if at all) from one planning effort to another.

(7) White papers are often used to describe how a map, database, or other product was developed. In this situation, the white paper functions as a 'user's guide' for the new product. Examples include papers dealing with historical products: (a) historical fire extents for the Tucannon watershed (WP Silv-21); (b) an 1880s map developed from General Land Office survey notes (WP Silv-41); and (c) a description of historical mapping sources (24 separate items) available from the Forest's history website (WP Silv-23).

These papers are available from the Forest's website: Silviculture White Papers

Paper #	Title
1	Big tree program
2	Description of composite vegetation database
3	Range of variation recommendations for dry, moist, and cold forests
4	Active management of Blue Mountains dry forests: Silvicultural considerations
5	Site productivity estimates for upland forest plant associations of Blue and Ochoco Mountains
6	Blue Mountains fire regimes
7	Active management of Blue Mountains moist forests: Silvicultural considerations
8	Keys for identifying forest series and plant associations of Blue and Ochoco Mountains
9	Is elk thermal cover ecologically sustainable?
10	A stage is a stage is a stage…or is it? Successional stages, structural stages, seral stages
11	Blue Mountains vegetation chronology
12	Calculated values of basal area and board-foot timber volume for existing (known) values of canopy cover
13	Created opening, minimum stocking, and reforestation standards from Umatilla National Forest Land and Resource Management Plan
14	Description of EVG-PI database
15	Determining green-tree replacements for snags: A process paper
16	Douglas-fir tussock moth: A briefing paper
17	Fact sheet: Forest Service trust funds
18	Fire regime condition class queries
19	Forest health notes for an Interior Columbia Basin Ecosystem Management Project field trip on July 30, 1998 (handout)
20	Height-diameter equations for tree species of Blue and Wallowa Mountains
21	Historical fires in headwaters portion of Tucannon River watershed
22	Range of variation recommendations for insect and disease susceptibility
23	Historical vegetation mapping
24	How to measure a big tree
25	Important Blue Mountains insects and diseases
26	Is this stand overstocked? An environmental education activity
27	Mechanized timber harvest: Some ecosystem management considerations
28	Common plants of south-central Blue Mountains (Malheur National Forest)
29	Potential natural vegetation of Umatilla National Forest

Paper #	Title
30	Potential vegetation mapping chronology
31	Probability of tree mortality as related to fire-caused crown scorch
32	Review of "Integrated scientific assessment for ecosystem management in the interior Columbia basin, and portions of the Klamath and Great basins" – Forest vegetation
33	Silviculture facts
34	Silvicultural activities: Description and terminology
35	Site potential tree height estimates for Pomeroy and Walla Walla Ranger Districts
36	Stand density protocol for mid-scale assessments
37	Stand density thresholds related to crown-fire susceptibility
38	Umatilla National Forest Land and Resource Management Plan: Forestry direction
39	Updates of maximum stand density index and site index for Blue Mountains variant of Forest Vegetation Simulator
40	Competing vegetation analysis for southern portion of Tower Fire area
41	Using General Land Office survey notes to characterize historical vegetation conditions for Umatilla National Forest
42	Life history traits for common Blue Mountains conifer trees
43	Timber volume reductions associated with green-tree snag replacements
44	Density management field exercise
45	Climate change and carbon sequestration: Vegetation management considerations
46	Knutson-Vandenberg (K-V) program
47	Active management of quaking aspen plant communities in northern Blue Mountains: Regeneration ecology and silvicultural considerations
48	Tower Fire…then and now. Using camera points to monitor postfire recovery
49	How to prepare a silvicultural prescription for uneven-aged management
50	Stand density conditions for Umatilla National Forest: A range of variation analysis
51	Restoration opportunities for upland forest environments of Umatilla National Forest
52	New perspectives in riparian management: Why might we want to consider active management for certain portions of riparian habitat conservation areas?
53	Eastside Screens chronology
54	Using mathematics in forestry: An environmental education activity
55	Silviculture certification: Tips, tools, and trip-ups
56	Vegetation polygon mapping and classification standards: Malheur, Umatilla, and Wallowa-Whitman National Forests
57	State of vegetation databases for Malheur, Umatilla, and Wallowa-Whitman National Forests
58	Seral status for tree species of Blue and Ochoco Mountains

REVISION HISTORY

October 2011: First version of this white paper was prepared for use with Mr. Kevin Steinmetz's elementary school class in John Day, Oregon. It had minor revisions many times since 1990 as it was used for other environmental education activities, such as outdoor schools. It was designed to show students how mathematics skills are used in one particular profession (forestry in this instance). For the October 2011 revision, it was modified to add an appendix describing a white-paper system, and a white-paper header and formatting was also implemented.